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Modified Gauss–Newton Algorithm for Evaluation of Full Lightning Impulse Voltage Parameters

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This paper presents an efficient curve fitting technique based on a Gauss–Newton (GN) method with a varied damping factor for the evaluation of lightning impulse voltage parameters. The proposed method can be used to determine the base curves of full lightning impulse voltages with oscillations corrected from the parameters of test cases according to the standard (IEC 61083-2). To verify the performance of the proposed method, base curves reconstructed from the developed method were compared with curves reconstructed from nonlinear least squares regression using the Levenberg–Marquardt (LM) algorithm, as prescribed in the standard. The proposed method provides the same results as the standard recommended method but its execution time is significantly shorter. The results of this study indicate that the proposed method is suitable for evaluating lightning impulse voltage parameters.

1. Introduction

Lightning overvoltage is a major cause of insulation failures of high-voltage equipment installed in high-voltage transmission and distribution systems. Therefore, it is necessary to perform lightning impulse voltage tests on high-voltage equipment to confirm its insulation performance. The lightning impulse voltage generated in a testing laboratory must have the waveform parameters required in the standard.[1,2]

A test object with a complicated structure under an impulse voltage test sometimes cannot be represented well by simple lumped circuit elements. Also, some parasitic circuit components in a real circuit, i.e., parasitic capacitance and inductance, lead to voltage waveforms with some oscillation and overshoot.[3–12] Therefore, it is necessary that a standard approach is defined for evaluating lightning impulse voltage waveform parameters.[1,2] The standard approach is based on determining the base curve and k-factor filtering to reduce the unwanted noise signal. By using digital waveform data and software for parameter evaluation, the standard (IEC 61083-2)[13] provides a software program named Test Data Generation (TDG) for the generation of

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waveforms, which are utilized to test the software for evaluating the impulse waveform parameters. In addition, the standard recommends nonlinear regression [Levenberg–Marquardt (LM) algorithm] to determine the base curve to evaluate the waveform parameters. However, nonlinear regression sometimes has low efficiency for a waveform with a high overshoot rate. This has motivated researchers to develop algorithms to increase the efficiency of nonlinear regression and also special linear regression algorithms for evaluating waveform parameters.[14–16]

In this paper, a Gauss–Newton (GN) method with a varied damping factor is proposed for evaluating the base curve of a full wave lightning impulse voltage. A comparison of the performance of the proposed method and the standard recommended method is presented. The proposed method shows reasonably high accuracy and noise immunity comparable to those of the standard recommended method but has a shorter computation time due to the lower complexity of the calculation.

2. Proposed Curve Fitting Method

For a better understanding of the proposed method, the waveforms in Fig. 1 are presented. According to the standard, the lightning impulse voltage waveform parameters can be evaluated by the following procedures.

(1) Collect the waveform from 20% of the peak voltage in the front part to 40% of the peak voltage in the tail part. This collected waveform is called the recorded waveform.

(2) Fit the recorded waveform with the function in Eq. (1). The coefficients of the function, i.e., α, β, A, and t_d, are computed, and the waveform with these coefficients is called the base curve.

$$g(t) = A(e^{-\alpha(t-t_d)} - e^{-\beta(t-t_d)}) \tag{1}$$

(3) Subtract the base curve from the recorded curve. The obtained result is the residual curve. Then, the residual curve is filtered by the k-factor filter, whose equation is expressed by

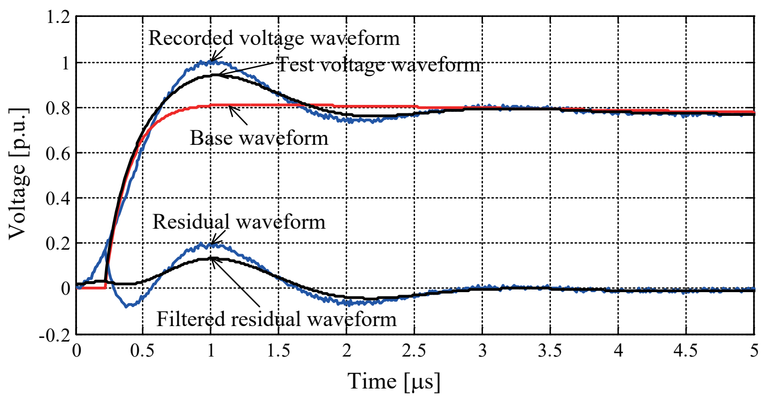

Fig. 1. (Color online) Process used to evaluate impulse voltage and current waveforms.

$$k(f) = \frac{1}{(1+2.2f^2)}, \tag{2}$$

where f is the frequency in MHz.

Lewin *et al.*[17] proposed an approach to implement the filter in the frequency domain in the form of a zero-phase-shift infinite impulse response (IIR) filter. The filtered waveform is called the filtered residual curve.

(4) Add the filtered residual curve to the base curve. The resulting waveform is called the test voltage curve. The waveform parameters, i.e., the front time (T_1), the time to half (T_2), the peak voltage (U_p), and the overshoot rate (β_e), are evaluated as percentages from the test voltage curve.

In this paper, a GN method with varied damping factors for obtaining a base curve is proposed as follows. Firstly, the recorded waveform is collected from 20% of the peak voltage in the front part to 40% of the peak voltage in the tail part. The time scale of this collected waveform is moved to start at zero. Then, the waveform is normalized with the offset level and the peak voltage.[15,16] The fitting function recommended by the standard method can be rearranged into the following simplified form:

$$f(t) = Ae^{-\alpha t} + Be^{-\beta t}. \tag{3}$$

Then, the four remaining parameters, A, B, α, and β, are estimated.

The GN method is a type of nonlinear regression or nonlinear curve fitting. For the sake of understanding, let us consider the square error function [$e(A, B, \alpha, \beta)$] defined as

$$e(A,B,\alpha,\beta) = \sum_{i=1}^{n}(y(i) - g(i,A,B,\alpha,\beta))^2, \tag{4}$$

where $y(i)$ is the value of the recorded waveform at point i.

The error function is minimized by the GN method to obtain the values of A, B, α, and β. The GN method starts from the approximation of $G(X)$ expressed as

$$G(X+\Delta X) \approx G(X) + J\Delta X, \tag{5}$$

where

$$X = [A\ B\ \alpha\ \beta]^t, \tag{6}$$

$$\Delta X = [\Delta A\ \Delta B\ \Delta\alpha\ \Delta\beta]^t, \tag{7}$$

$$G(X) = [g(1,X)\ g(2,X)\ g(3,X) \ldots g(n,X)]^t\,, \tag{8}$$

$$J = \begin{bmatrix} \dfrac{\partial g(1,X)}{\partial A} & \dfrac{\partial g(1,X)}{\partial B} & \dfrac{\partial g(1,X)}{\partial \alpha} & \dfrac{\partial g(1,X)}{\partial \beta} \\ \dfrac{\partial g(2,X)}{\partial A} & \dfrac{\partial g(2,X)}{\partial B} & \dfrac{\partial g(2,X)}{\partial \alpha} & \dfrac{\partial g(2,X)}{\partial \beta} \\ \dfrac{\partial g(3,X)}{\partial A} & \dfrac{\partial g(3,X)}{\partial B} & \dfrac{\partial g(3,X)}{\partial \alpha} & \dfrac{\partial g(3,X)}{\partial \beta} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial g(n,X)}{\partial A} & \dfrac{\partial g(n,X)}{\partial B} & \dfrac{\partial g(n,X)}{\partial \alpha} & \dfrac{\partial g(n,X)}{\partial \beta} \end{bmatrix}. \tag{9}$$

Equation (5) can be rewritten as Eq. (10). Assuming that $G(X + \Delta X) = Y$ and applying linear regression to Eq. (10), we obtain ΔX as

$$E(X + \Delta X) = G(X + \Delta X) - G(X) = J\Delta X \tag{10}$$

$$\Delta X = (J^t J)^{-1} J^t E(X + \Delta X)\,. \tag{11}$$

To obtain the minimum value of the error function, X is updated to $X + \Delta X$ and is calculated iteratively until the relative error (ε_i) is less than the tolerance limit defined by Eq. (12). In this paper, the tolerance limit is set to 10^{-6}. ε_i is given by

$$\varepsilon_i = \frac{\| e(A,B,\alpha,\beta)_i - e(A,B,\alpha,\beta)_{i-1} \|}{\| e(A,B,\alpha,\beta)_{i-1} \|}, \tag{12}$$

where ε_i is the relative error at the ith iteration. The maximum number of iterations in the curve fitting method is set to 200.

It has been found that when the conventional GN method is applied to evaluate the waveform parameters using the standard waveforms,[13] the solution of X sometimes cannot reach the set tolerance limit. To solve this problem, the updated value of X is set to

$$X_{k+1} = X_k + d\Delta X_k\,, \tag{13}$$

where X_{k+1} is the updated value of X at the kth iteration and d is the damping factor.

3. Performance of Proposed Method

In this section, 29 standard waveforms taken from the standard[13] are used in parameter evaluation. The time step and the resolution of all waveforms were set to 10 ns and 12 bits, respectively. The standard recommended and proposed methods were applied to evaluate the waveform parameters. In the proposed method, the damping factor (d) is considered. It is found that the GN method with fixed d (first option) and with d varied during the iteration (second option) can obtain the same waveform parameters as the standard recommended method (LM). The waveform parameters are presented in Fig. 2.

Fig. 2. (Color online) Waveform parameters evaluated by the proposed method and the standard recommended method in comparison with the standard tolerances. (a) test voltage, (b) front time, (c) time to half, and (d) overshoot rate.

In the first option, the optimal value of the constant damping factor is found by testing with d varied from 0.5 to 1.2. When $d > 1.0$, the GN method cannot reach the optimum solution after 200 iterations in some test cases, making it unsuitable for waveform parameter evaluation. In contrast, the GN method with $d < 1.0$ can find the optimum solution in all test cases. However, when $d < 0.8$, a large number of iterations (a few tens of iterations) and a long execution time are required to obtain the parameters by the GN method. The GN method with $d = 0.8$ provides the best performance in terms of the execution time and the number of iterations.

On the basis of the results for the first option of the damping factor, we decided to vary the damping factor during the iteration process (second option). d is set to 1 for the first and second iterations and to 0.8 for the other iterations. The parameters of the normalized base waveforms calculated by the LM and GN methods are the same, as shown in Table 1. It was found that the performance of this proposed method in terms of the execution time is superior to that of the GN method with a constant damping factor, as can be seen from Table 2. The GN method with the varied damping factor can find the optimum solution and the waveform parameters in all test cases. The average number of iterations and the execution time of the GN method with the varied

Table 1
Parameters of the base waveforms computed by the GN and LM methods.

Case	A	B	α	β
LI-A1	1.044	−0.817	1.193E4	3.614E6
LI-A2	0.998	−0.826	1.476E4	1.932E6
LI-A3	0.978	−0.905	1.475E4	2.523E6
LI-A4	0.928	−0.867	1.377E4	4.034E6
LI-A5	0.968	−0.900	1.406E4	1.985E6
LI-A6	0.865	−0.809	1.336E4	2.576E6
LI-A7	0.844	−0.785	1.377E4	2.295E6
LI-A8	0.884	−0.827	1.307E4	2.689E6
LI-A9	0.989	−0.899	1.240E4	2.568E6
LI-A10	0.888	−0.847	1.463E4	4.008E6
LI-A11	0.986	−0.874	1.216E4	5.433E6
LI-A12	1.002	−0.902	1.213E4	5.208E6
LI-M1	0.998	−0.784	8.263E3	2.660E6
LI-M2	0.950	−0.770	1.134E4	1.041E6
LI-M3	0.964	−0.786	1.709E4	1.559E6
LI-M4	0.975	−0.848	1.203E4	3.005E6
LI-M5	0.870	−0.774	1.284E4	1.689E6
LI-M6	1.008	−0.830	1.300E4	2.192E6
LI-M7	0.932	−0.902	1.268E4	2.308E6
LI-M8	1.055	−0.883	1.522E4	1.926E6
LI-M9	1.022	−0.893	1.532E4	3.572E6
LI-M10	1.051	−0.670	1.234E4	1.572E6
LI-M11	1.059	−0.830	1.229E4	1.702E6
LI-M12	1.067	−0.860	1.454E4	2.142E6
LI-M13	1.027	−0.874	1.554E4	1.948E6
LI-M14	1.003	−0.894	1.880E4	3.324E6
LI-M15	1.032	−0.924	1.232E4	2.802E6
LI-M16	1.044	−0.823	1.524E4	3.217E6
LI-M17	1.038	−0.704	1.415E4	1.572E6

Table 2
Comparison of performance of the standard recommended and proposed methods.

Case	LM		GN (d = 1.0)		GN (d = 0.8)		GN (varied d)	
	Number of iterations	Execution time (s)	Number of iterations	Execution time (s)	Number of iterations	Execution time (s)	Number of iterations	Execution time (s)
LI-A1	5	0.02126	4	0.01073	7	0.01474	6	0.01396
LI-A2	6	0.03636	6	0.00927	6	0.00605	4	0.01119
LI-A3	4	0.01136	4	0.00579	6	0.01154	4	0.00952
LI-A4	10	0.02483	9	0.01404	6	0.00863	6	0.00892
LI-A5	10	0.02427	9	0.01515	6	0.00753	6	0.01033
LI-A6	30	0.05532	28	0.03013	6	0.02199	7	0.00744
LI-A7	13	0.02800	200 (F)	0.24233	10	0.02516	13	0.01713
LI-A8	18	0.04165	18	0.01913	6	0.01002	8	0.00933
LI-A9	5	0.01531	5	0.01279	6	0.00823	4	0.00714
LI-A10	13	0.02645	12	0.01728	7	0.01094	7	0.00676
LI-A11	6	0.01568	5	0.00928	7	0.01631	6	0.01289
LI-A12	7	0.01984	6	0.01108	7	0.00975	6	0.01197
LI-M1	6	0.02671	6	0.00966	7	0.01576	7	0.01213
LI-M2	7	0.01996	9	0.01014	7	0.00938	9	0.00810
LI-M3	6	0.01594	6	0.00756	7	0.00770	6	0.00710
LI-M4	4	0.00982	4	0.00441	6	0.00777	4	0.00537
LI-M5	59	0.09794	57	0.03918	11	0.01123	13	0.01508
LI-M6	4	0.01172	4	0.00742	6	0.01417	4	0.00720
LI-M7	12	0.02648	12	0.00809	6	0.00756	7	0.00841
LI-M8	4	0.01318	4	0.00787	6	0.00587	4	0.00795
LI-M9	5	0.01321	5	0.00551	6	0.00469	5	0.00549
LI-M10	4	0.00983	4	0.00996	7	0.00664	5	0.01284
LI-M11	4	0.03606	4	0.00812	7	0.01300	5	0.01332
LI-M12	5	0.01295	4	0.01078	7	0.01061	5	0.00861
LI-M13	6	0.01446	5	0.00742	6	0.03880	5	0.00791
LI-M14	6	0.01186	6	0.00706	7	0.00909	5	0.02297
LI-M15	4	0.01445	4	0.00825	7	0.01108	4	0.00934
LI-M16	5	0.01563	4	0.01062	6	0.01965	5	0.00916
LI-M17	4	0.01160	4	0.00944	8	0.01397	4	0.01488
Average	9.38	0.02352	15.45	0.01960	6.79	0.01234	6.00	0.01043
Max	59	0.09794	200	0.24233	11	0.03880	13	0.02297
Min	4	0.00982	4	0.00441	6	0.00469	4	0.00537
Median	6	0.01594	5	0.00966	7	0.01061	5	0.00933

Note that the GN method with d = 1.0 failed to determine the optimum solution in case LI-A7.

damping factor are slightly less than those of the GN method with the constant damping factor and the standard recommended method, but the execution time of the proposed method is only half that of the LM algorithm. In addition, the LM algorithm is more complicated than the proposed method, which can be readily implemented as a program for evaluating the lightning waveform parameters.

The performances of the methods in terms of the number of iterations and the execution time are compared in Table 2. Owing to space limitation, only the performances of the cases with d = 1, d = 0.8, and d = 1 for the first and second iterations and d = 0.8 thereafter are presented.

Figures 3–5 illustrate the results for cases LI-A7, LI-M5, and LI-M17, respectively. The blue solid lines, black solid lines, and red dotted lines denote the recorded, test, and base waveforms, respectively.

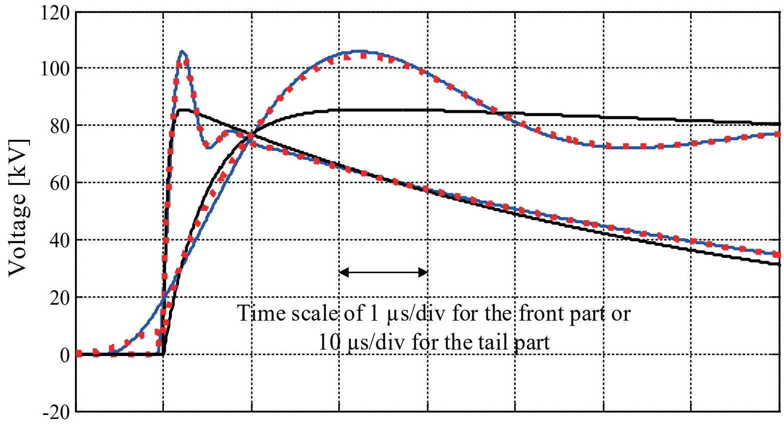

Fig. 3. (Color online) Waveforms evaluated by the proposed method in case LI-A7.

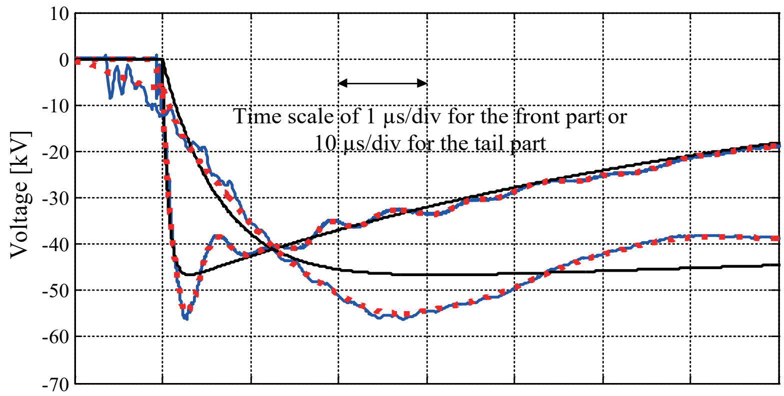

Fig. 4. (Color online) Waveforms evaluated by the proposed method in Case LI-M5.

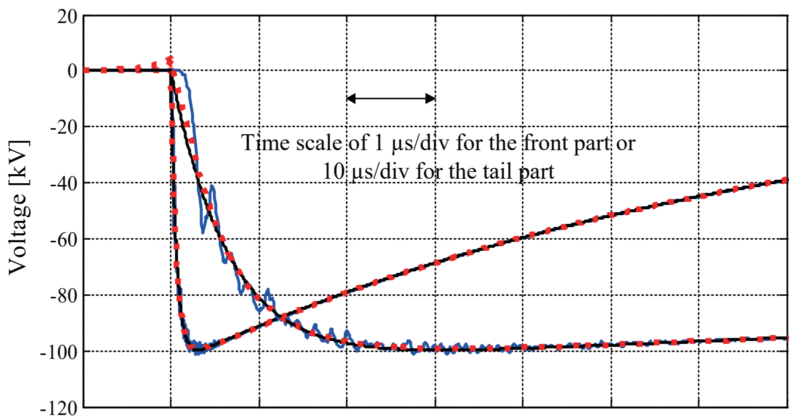

Fig. 5. (Color online) Waveforms evaluated by the proposed method in Case LI-M17.

4. Conclusions

An efficient curve fitting technique based on the GN method with the damping factor varied was proposed for the evaluation of the lightning impulse voltage parameters. The validity of the proposed method was verified using the standard waveforms collected from the test cases according to the standard (IEC 61083-2). The proposed method provides the same results as the standard recommended method (LM algorithm). The performances of the proposed method in terms of the number of iterations and the execution time were investigated in comparison with the standard recommended method. It was found that the number of iterations and the execution time of the proposed method are significantly less than those of the GN method with a constant damping factor and the standard recommended method. Owing to the low complexity of the model, the proposed method is very attractive for the development of software to evaluate the full lightning waveform parameters by test engineers and scientists lacking experience in software development.

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