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Accurate Estimation of Number of Signal Sources by Eigenvalue Quadratic Diagonal Loading

Chuanchuan Wang, Yonghu Zeng,* Zhipeng Li, and Liandong Wang

State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System, Luoyang 471003, China

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To solve the problem of estimating the number of sources when an antenna array is used for signal reception, an eigenvalue quadratic diagonal loading technique is proposed. The proposed method is applicable under the condition of the proportional relationship between the number of antenna elements and that of snapshots, and whether the received signals are mixed with white Gaussian or colored noise is unknown. The method we proposed belongs to an important area of antenna array signal processing in the field of sensor technology. Firstly, the covariance matrix of received signals is eigenvalue-decomposed. Secondly, eigenvalues are loaded diagonally. The first diagonal loading value is taken as the arithmetic average of all eigenvalues, and the original eigenvalues and diagonal loading value are added to replace the original eigenvalues. The quadratic diagonal loading formula is devised to carry out the quadratic diagonal loading on the eigenvalues immediately after the first diagonal loading. Finally, the information theoretic criterion and random matrix theory methods are chosen to combine with the proposed method to estimate the number of sources. The method is validated by simulation experiments. By combining the proposed method with the existing information theoretic criterion methods, the proposed method can be suitable for the general asymptotic regime with the same number of antenna elements and snapshots in a white Gaussian or colored noise environment. The application range of the random matrix theory methods is extended by the proposed method, so that it can be applied to the colored noise environment.

1. Introduction

The estimation of number of sources has important applications in many fields, such as phased array radar,[1,2] brain imaging,[3] speech signal separation,[4] and the direction of arrival (DOA) estimation.[5] For example, as to the problem of DOA estimation, most of the algorithms need the number of sources as an input parameter. In multiple-input multiple-output (MIMO) radar signal processing, multitarget parameter estimation and localization are hot research

*Corresponding author: e-mail: duoduo17wang@163.com
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fields. However, their prerequisite is to detect the number of targets in a certain noise environment. In the domain of speech signal separation by blind source separation methods, the correct estimation of the number of persons is crucial for separation effects.

The methods for estimating the number of sources are essentially based on the statistical analysis theory of observed data and their moment functions. For example, the hypothesis testing and information theoretic criterion (ITC) methods, which mainly use the statistical distribution of observed data and the statistics of sample eigenvalues,[6] are commonly used for estimating the number of sources. At present, estimation methods are mainly based on the classical asymptotic system, whose dimension of the observed data matrix is fixed and the number of snapshots tends to be infinite. These methods are suitable for small-scale array signals whose sample number is much larger than that of antenna elements.[7]

However, in large-scale sensor arrays such as phased array radar and MIMO systems, owing to the limitation of data storage space and the real-time requirement of signal processing, it is often difficult for the observed data to meet the condition that the number of snapshots is much larger than that of elements, which usually belongs to high-dimensional limited or even small sampling data. The number of snapshots is of the same order of magnitude as that of elements, or even less than the number of elements. As to large-scale array observed data, the proportional relationship between the number of snapshots and that of elements often does not meet the requirements of the classical statistical theory. Therefore, the emergence of a large-scale array raises new challenges to classical estimation methods of the number of sources.[8,9]

At present, among the estimation methods of the number of sources under the classical asymptotic system, hypothesis testing methods include spherical test[10] and eigenvalue detection,[11] which are mainly used to construct the observation statistics for hypothesis testing and to set the decision threshold by using the statistical distributions law of sample eigenvalues. ITC methods include Akaike information criterion (AIC),[12] Bayesian information criterion (BIC),[13] minimum description length (MDL),[14] and predictive description length (PDL).[15] The observed data are Gaussian distributions, which establish a criterion for estimating the number of sources according to the likelihood function of the joint probability distribution of the observed data. The expression of estimating the number of sources is a function of the sample eigenvalues. These methods are suitable for a white Gaussian noise environment. In the classical asymptotic system, the main methods for estimating the number of sources in colored noise are the Gerschgorin circle method[16] and ITC methods based on diagonal loading,[2,17] but these methods are not suitable for large-scale arrays.

Estimation of the number of sources in the general asymptotic regime is mainly based on the random matrix theory (RMT), including the RMT-AIC method,[18] B. Nadler-AIC (BN-AIC) method,[19] BIC-variant method,[20] linear shrinkage-MDL (LS-MDL) method,[21] and the method based on a spike model,[22] which are applicable when the number of elements is less than that of snapshots. For the estimation methods based on the spherical test and corrected Rao's score test,[22] they are applicable when the number of elements is more than, less than or equal to the number of snapshots. All these methods are suitable not only for estimating the number of sources in the general asymptotic regime, but also for the classical asymptotic system. However, these methods are only applicable to a white Gaussian noise environment, and they fail in a colored noise environment.

In practice, the signals received by the array antenna contain complex spatial colored noises. In the colored noise environment, the noise eigenvalues of the covariance matrix of the received signals will become very divergent, and the noise eigenvalues will not vibrate near the noise power as they do in white Gaussian noise. This result caused by colored noise will invalidate various algorithms for estimating the number of sources using hypothesis testing and ITC methods. The estimation method of the number of sources based on the Gerschgorin theorem and the methods based on eigenvalue diagonal loading combined with ITC methods can only be applied to the classical asymptotic system, that is, the relationship between the number of antenna elements, M, and that of signal snapshots, N, is M fixed and $M/N \ll 1$. In the following, M denotes the number of antenna elements, which is also the dimension of observed signals $\boldsymbol{X}(t)$. N denotes the number of snapshots of signals. In the general asymptotic regime, that is, the relationship between the number of antenna elements, M, and that of snapshots, N, is that M and N tend to infinity at the same rate, i. e.,$M, N \to \infty$ and $M/N \to c \in (0, \infty)$. A new estimation method based on the Gerschgorin circle transform and generalized Bayesian information criterion is devised,[23] in the case that the observed signals are overlapped with colored noise, and the number of elements compared with that of snapshots meets the requirement of the general asymptotic regime. However, the estimation method for the number of sources suitable for a colored noise environment is insufficient.

In such a case, an eigenvalue quadratic diagonal loading method for estimating the number of sources is developed in this study. By combining the proposed method with the ITC methods, the ITC methods are extended to general asymptotic regimes, and the noise environment can be white Gaussian or colored noise. At the same time, this method extends the application of the RMT methods, which can be applied to colored noise. Compared with the existing eigenvalue diagonal loading methods, the proposed method adopts the eigenvalue quadratic diagonal loading, which is equivalent to the second correction of the eigenvalue distribution, and can realize the estimation of the number of sources more robustly.

The technique that we studied is used in passive or active detection sensor systems, specifically the advanced technique of array signal processing using antenna arrays. There is no doubt that the antenna array and receiver are important in the field of sensor technology. The remainder of this paper is organized as follows. In Sect. 2, we present the model of estimating the number of sources problem. In Sect. 3, we discuss the main basis of the proposed method. In Sect. 4, we give the proposed estimation method of the number of sources. In Sect. 5, we describe the experimental results that illustrate the effectiveness of the proposed method. Finally, the conclusions are given in Sect. 6.

2. Mathematical Model of Estimating the Number of Sources

Suppose there are far-field signals whose number is K incidenting from the directions $\theta_1, \theta_2, \cdots, \theta_K$ onto an array antenna, and the number of elements is M. At the sampling time t, the expression of the observed signals of the antenna is shown in Eq. (1),

$$\boldsymbol{X}(t)=\sum_{k=1}^{K}a(\theta_k)s_k(t)+\boldsymbol{w}(t)=\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t)+\boldsymbol{w}(t), \tag{1}$$

where $\boldsymbol{X}(t)=\left[X_1(t),X_2(t),\ldots,X_M(t)\right]^T$ (the superscript T represents transpose) is the observed signal vector, $a(\theta_k)$ is the array direction vector, $\boldsymbol{A}(\boldsymbol{\theta})=\left[a(\theta_1),a(\theta_2),\cdots,a(\theta_K)\right]$ is the matrix composed of direction vectors, $\boldsymbol{\theta}=\left[\theta_1,\theta_2,\cdots,\theta_K\right]^T$ is the incoming wave angle parameter vector of the signals, $\boldsymbol{s}(t)=\left[s_1(t),s_2(t),\cdots,s_K(t)\right]^T$ is the incident signal vector, $\boldsymbol{w}(t)=\left[w_1(t),w_2(t),\cdots,w_M(t)\right]^T$ is the additive noise vector, $t=1, 2, \cdots, N$ is the sampling time, and N is the number of snapshots. The basic assumptions of the array observed signal model shown in Eq. (1) are as follows:(23)

(1) The incident signals are narrowband stationary signals independent of each other, which satisfy the mean $E\{\boldsymbol{s}(t)\}=0$ and covariance matrix $E\left\{\boldsymbol{s}(t)\boldsymbol{s}^H(t)\right\}=diag\left\{p_{s_1},p_{s_2},\cdots,p_{s_K}\right\}\triangleq P_s\in\mathbb{R}^{K\times K}$, where p_{s_K} is the power of the k-th source signal.
(2) The superimposed noise in the observed signal vector is additive noise (white Gaussian or colored noise), which is independent of the incident signals.
(3) The number of incident signals is less than those of antenna elements and snapshots at the same time, that is, $K<\min(M,N)$.
(4) The incident signals propagate in ideal space, and the antenna elements have omnidirectional consistency.

3. Main Basis of Proposed Method

The method proposed in this paper is mainly based on the following propositions:

Proposition 1: According to the estimation methods of the number of sources based on ITC, when the ratio of the maximum to minimum noise eigenvalues in the observed signals of the array antenna is less than 2, the ITC methods can correctly estimate the number of sources,(24) that is,

$$\frac{\lambda_{K+1}}{\lambda_M}<2, \tag{2}$$

where M and K are the number of antenna elements and that of signals, and λ_{K+1} and λ_M are the maximum and minimum noise eigenvalues, respectively.

Proposition 2: For the eigenvalues of the covariance matrix of the observed signals of the array antenna, it is found that the noise eigenvalues are very divergent in the colored noise environment, and diagonal loading can reduce the divergence of the noise eigenvalues and make them close to equality, and it does not have a significant effect on some of the eigenvalues of the signals.(25–28)

The eigenvalues of the covariance matrix are calculated as $\boldsymbol{R}(t)=\boldsymbol{X}(t)\cdot\boldsymbol{X}^H(t)/N$, as do the eigen-decomposition to the covariance matrix $\boldsymbol{R}(t)$, and we can obtain the eigenvalues $\{\lambda_k\}_{k=1}^{M}$, which are numerical sequences arranged in descending order. By diagonal loading, the eigenvalues $\{\lambda_k\}_{k=1}^{M}$ are changed as

$$\lambda_k^{'} = \lambda_k + \lambda_{DL}, \tag{3}$$

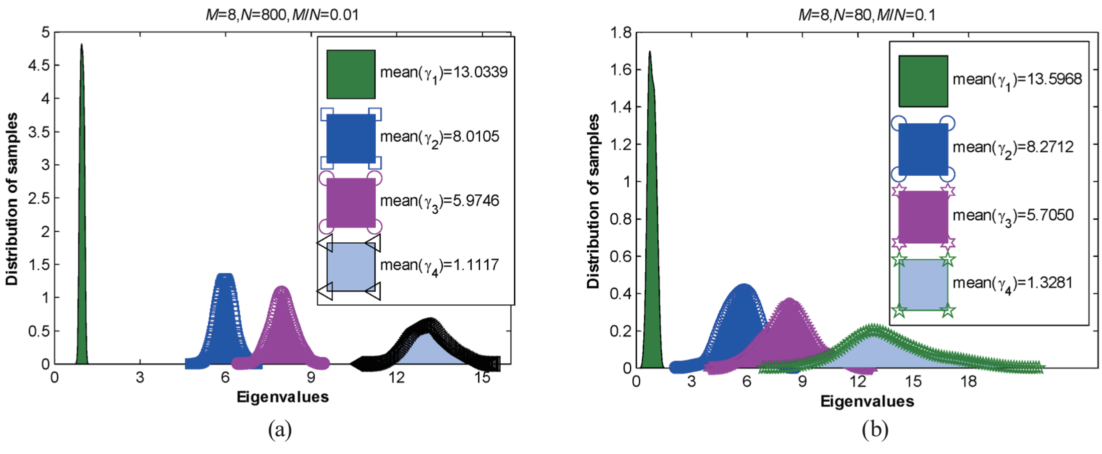
where λ_{DL} is the diagonal loading.

However, the difficulty of the eigenvalue diagonal loading is that there is no theoretical formula for calculating it, and only some empirical formulas are available. If the eigenvalue diagonal loading is very small, it is impossible to overcome the effect of nonuniform noise on the eigenvalues. On the contrary, overloading occurs, which results in the under estimation of the number of sources.[25–28]

Proposition 3: When the number of snapshots is not very large compared with that of antenna elements, both the eigenvalues of signals and sample noise are biased estimates of real values, and the fusion of the two types of eigenvalue cannot be separated, which leads to errors in the estimation of the number of sources by ITC methods.[23]

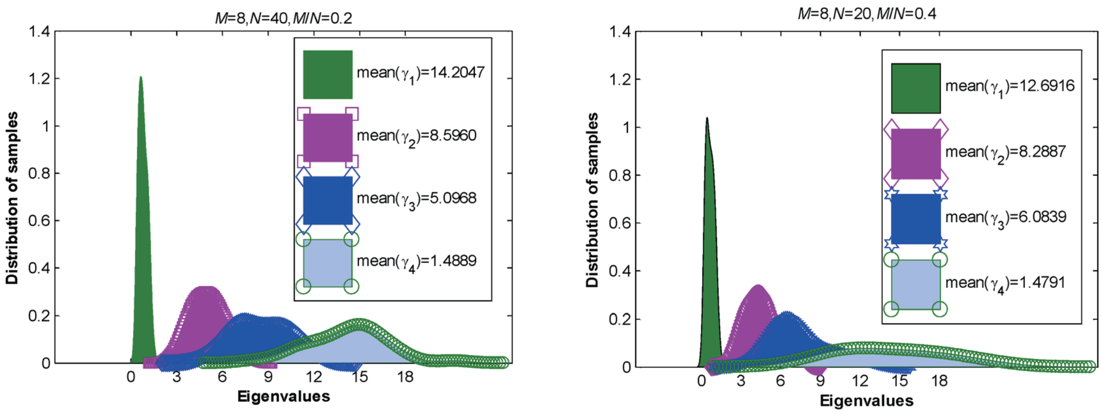
The effects of the change in the proportional relationship between the number of antenna elements and that of snapshots on the signal and noise eigenvalues are analyzed as follows.

The expressions of estimation of the number of sources by ITC methods are functions of sample eigenvalues. The statistical distribution of signal eigenvalues is studied through simulation experiments, assuming that the total covariance matrix is $\mathbf{\Sigma} = diag\{13,8,6,1,1,1,1,1\}$, and the observation sample matrix is set as $\boldsymbol{Z}_N = \sqrt{\mathbf{\Sigma}}\boldsymbol{X}_N$, where $\boldsymbol{X}_N$ is an $M \times N$ dimensional Gaussian random matrix (here, $M = 8$), and the probability distribution of its elements $x_{i,j}$ obeys $\mathbb{N}(0,1)$, here $i = 1, 2, \cdots, M$, and $j = 1, 2, \cdots, N$. Five hundred independent repeated experiments were carried out, the distribution of eigenvalues of sample covariance matrix $\hat{\mathbf{\Sigma}} = \boldsymbol{Z}_N \boldsymbol{Z}_N^H / N$ was determined, and the sample mean values of eigenvalues $\gamma_1 = 13$, $\gamma_2 = 8$, $\gamma_3 = 6$, and $\gamma_4 = 1$ were calculated. The statistical distribution of sample eigenvalues with different sample numbers is shown in Fig. 1. In Fig. 1(a), $M/N = 0.01$ satisfies $N \gg M$, the estimates of γ_1, γ_2, γ_3, and γ_4 can



(a) (b)

Fig. 1. (Color online) Distribution of sample eigenvalues of covariance matrix with different sample numbers.



(c) (d)

Fig. 1. (Color online) (Continued) Distribution of sample eigenvalues of covariance matrix with different sample numbers.

be clearly distinguished, and the mean values of estimates of γ_1, γ_2, γ_3, and γ_4 obtained from 500 simulation experiments are very close to the real eigenvalues, which are γ_1, γ_2, γ_3, and γ_4. In Fig. 1(b), M/N = 0.1, which means that M and N are of the same order of magnitude. The estimates of γ_1, γ_2, γ_3, and γ_4 are fused together, and the mean values of estimates of γ_1, γ_2, γ_3, and γ_4 obtained from 500 simulation experiments also deviate from the real eigenvalues. In Fig. 1(c), M/N = 0.2, and in Fig. 1(d), M/N = 0.4, M and N are of the same order of magnitude in the two cases, which also reflects that the estimates of γ_1, γ_2, γ_3, and γ_4 are fused together, and the mean values of estimates of γ_1, γ_2, γ_3, and γ_4 also deviate from the real eigenvalues.

To further reflect the above laws, seven estimation methods of the number of sources are used to carry out simulation experiments under different snapshots, which are AIC,[(12)] MDL,[(14)] AIC combined with eigenvalue diagonal loading (IAIC),[(7)] MDL combined with eigenvalue diagonal loading (IMDL),[(29)] nonuniform MDL (NUMDL),[(30)] ratio of singular value decomposition (RSVD),[(25)] and the method of improved BIC (MIBIC).[(7)] The signal-to-noise ratio (SNR) of observed signals varies from –10 to 30 dB, the step size is 5 dB, and 500 Monte Carlo simulations are carried out at each SNR. The experimental results are shown in Figs. 2(a)–2(d). As can be seen from Fig. 2(a), when M/N = 0.01 satisfies $N \gg M$, ideal estimation results of the number of sources can be obtained by a variety of algorithms at lower SNRs. From Figs. 2(b) and 2(c), we can see that when M/N = 0.1 or M/N = 0.2, a small number of algorithms can obtain ideal estimation results at high SNRs. In Fig. 2(d), it is difficult for all algorithms to obtain the correct estimation results.

From the AIC criterion expression, it can be seen that the estimation performance of the number of sources depends on the accuracy of sample eigenvalues. The ITC methods can accurately estimate the number of sources only when the eigenvalues of the sample signal $\hat{\lambda}_1, \cdots, \hat{\lambda}_K$ and the eigenvalues of the sample noise $\hat{\lambda}_{K+1}, \cdots, \hat{\lambda}_M$ can be well separated. The likelihood logarithm term in the AIC criterion expression is the ratio of the arithmetic mean value to the geometric mean value of the sample noise eigenvalues. The more concentrated the sample noise eigenvalues are and the farther they are from the sample signal eigenvalues, the

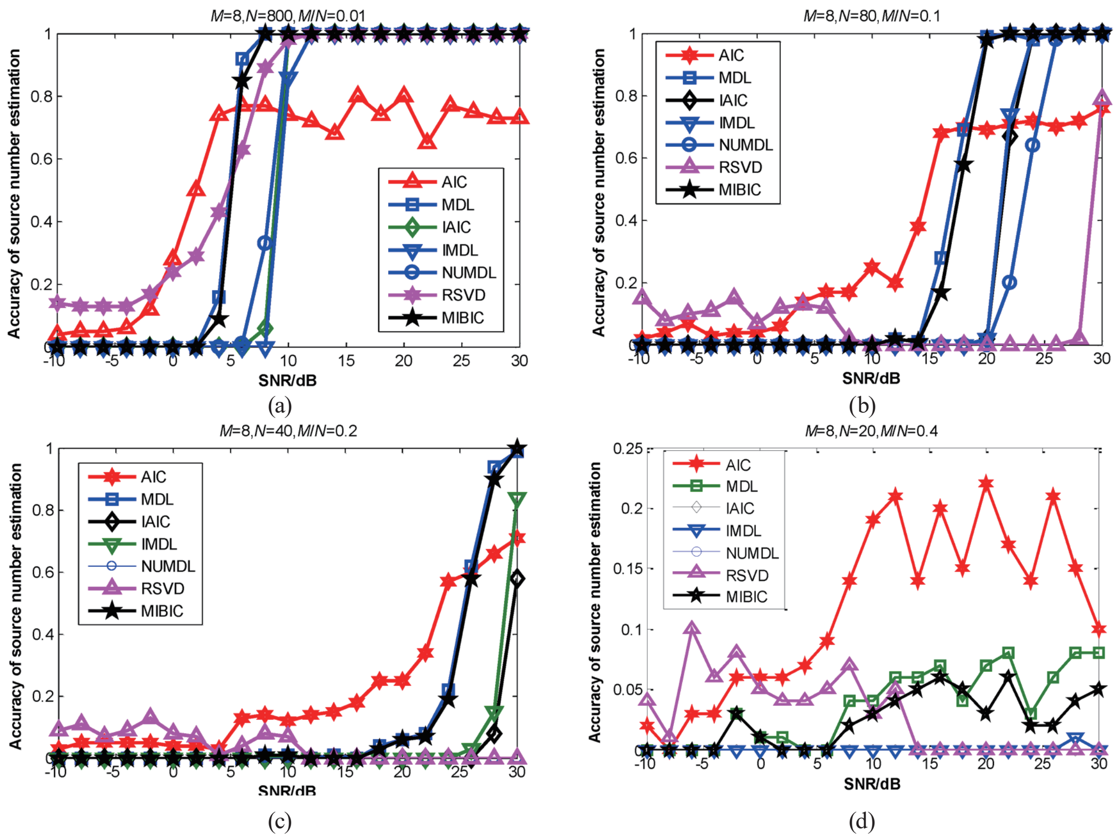


Fig. 2. (Color online) Estimation results of the number of sources with different sample numbers.

more accurate is the estimation of the number of sources. The distribution of sample eigenvalues is affected by the ratio of dimension to sample number, which is $c_N = M/N$, and the SNR. When $N \gg M$, the sample noise eigenvalues $\hat{\lambda}_{K+1}, \cdots, \hat{\lambda}_M$ are concentrated on σ^2, which is the power of noise, and are far away from the sample signal eigenvalues $\hat{\lambda}_1, \cdots, \hat{\lambda}_K$. When N compared with M is not very large, but at the same order of magnitude, the sample noise eigenvalues $\hat{\lambda}_{K+1}, \cdots, \hat{\lambda}_M$ are dispersed and fused with the signal eigevalues $\hat{\lambda}_1, \cdots, \hat{\lambda}_K$, resulting in the incorrect estimation of the number of sources. When $N < M$, the geometric mean value of the sample signal eigenvalues is equal to zero, which leads to the likelihood logarithm term in the AIC criterion becoming meaningless. Even if the $M - N$ zero sample eigenvalues are removed, the fusion of sample noise and signal eigenvalues also leads to a serious decline in the performance of the ITC methods.

4. Proposed Eigenvalue Diagonal Loading Method

Through our experiments, it is found that for the problem of estimation of the number of sources in the general asymptotic regime and colored noise environment, by controlling the divergence of noise eigenvalues to satisfy Proposition 1, the applicable domain of the existing estimation methods can be greatly expanded, including some ITC and RMT methods.

Inspired by Proposition 1–Proposition 3, aiming at the problem of estimation of the number of sources under the condition of observed signals mixed with colored noise in the general asymptotic regime, a quadratic diagonal loading method for eigenvalues is proposed. The idea is to perform the eigen-decomposition of the covariance matrix of observed signals, and then the eigenvalues are firstly loaded diagonally, and the first diagonal loading is taken as the arithmetic mean value of all eigenvalues. The original eigenvalues and the diagonal loading value are added to replace the original eigenvalues. For eigenvalues after the first diagonal loading, the new diagonal loading of the eigenvalue is recalculated, and the second diagonal loading is carried out for the eigenvalues after the first diagonal loading, in order to satisfy the condition that the ratio of the maximum to minimum values of the noise eigenvalues does not exceed 2. After these steps, the ITC and RMT methods can be employed to estimate the number of sources. The role of diagonal loading is as follows: when the noise eigenvalue is very divergent, the loading quantity controls its divergence, which is equivalent to "whitening" colored noise, and when the number of samples is not very large compared with that of antenna elements, it can also reduce the fusion degree of signal and noise eigenvalues, such that they are separated more clearly to facilitate the estimation of the number of sources.

The following are the specific steps of the proposed method:

Step 1: assume that the number of antenna elements is M and the observed signals can be expressed as $\boldsymbol{X}(t)=\left[X_1(t),X_2(t),\ldots,X_M(t)\right]^T$ (the superscript T represents transpose). The sampling time is $t=1,2,\cdots,N$, N is the number of snapshots, and the covariance matrix of the observed signals is calculated as $\boldsymbol{R}(t) = X(t)\cdot X^H(t) / N$.

Step 2: perform eigen-decomposition to the covariance matrix $\boldsymbol{R}(t)$, $\boldsymbol{R}(t)=\sum_{i=1}^{M}\lambda_i u_i u_i^H$, where the eigenvalues λ_i and the eigenvectors u_i are also respectively called sample eigenvalues and eigenvectors. The eigenvalue sequence is expressed as $\{\lambda_k\}_{k=1}^{M}$, which is a numerical sequence arranged in descending order.

Step 3: carry out the first diagonal loading for the eigenvalue sequence $\{\lambda_k\}_{k=1}^{M}$, and the formula for calculating the first diagonal loading is expressed as

$$jz1=\frac{1}{M}\sum_{k=1}^{M}\lambda_k. \tag{4}$$

Step 4: according to the sample covariance matrix $\boldsymbol{R}(t)$ and the eigenvalue first diagonal loading Eq. (4), the new sample covariance matrix after diagonal loading is calculated, which is expressed as

$$\tilde{\boldsymbol{R}}_1(t)=\boldsymbol{R}(t)+jz1\cdot\boldsymbol{I}_M, \tag{5}$$

where $\boldsymbol{I}_M$ is an M-dimensional unit matrix.

Step 5: perform eigen-decomposition to the covariance matrix $\tilde{\boldsymbol{R}}_1(t)$, then a new eigenvalue sequence $\{\tilde{\lambda}_k\}_{k=1}^{M}$ is obtained.

Step 6: carry out the second diagonal loading for the eigenvalue sequence $\{\tilde{\lambda}_k\}_{k=1}^{M}$. The determination of the second diagonal loading is as follows:

(1) Obtain the sequence number of the eigenvalue with the highest ratio of two consecutive eigenvalues:

For the sequence of eigenvalues $\{\tilde{\lambda}_k\}_{k=1}^{M}$ in descending order, select $\tilde{k} = \max_k \frac{\lambda_{k+1}}{\lambda_k}$, $k = 1, 2, \cdots, M-1$.

(2) Calculate the smallest integer $\hat{k}$ such that $\frac{\lambda_{\tilde{k}+1} + 0.1\times\hat{k}}{\lambda_M + 0.1\times\hat{k}} < 2$ holds:

$$\hat{k} = \begin{cases} \text{rounding up}\left(\dfrac{\lambda_{\tilde{k}+1} - 2\lambda_M}{0.1}\right), & \dfrac{M}{N} < 0.6, \\ \text{rounding up}\left(\text{absolute value of}\left(\dfrac{\lambda_{\tilde{k}+1} - 2\lambda_M}{0.1}\right)\right), & \dfrac{M}{N} \geq 0.6. \end{cases} \tag{6}$$

(3) Set the second diagonal loading quantity $jz2 = 0.1\times\hat{k}$, carry out the second diagonal loading for the eigenvalue sequence $\left\{\tilde{\lambda}_k\right\}_{k=1}^{M}$, and obtain a new eigenvalue sequence $\left\{\hat{\lambda}_k\right\}_{k=1}^{M}$:

$$\hat{\lambda}_k = \tilde{\lambda}_k + jz2, \quad k = 1, 2, \cdots, M. \tag{7}$$

Step 7: employ the ITC and RMT methods to estimate the number of sources based on the new eigenvalue sequence $\left\{\hat{\lambda}_k\right\}_{k=1}^{M}$.

5. Simulation Experiment and Analysis

The proposed algorithm is applied to a uniform linear array (ULA), the array element interval is half-wavelength, and the DOAs are one dimension.

The validation of the proposed method is carried out on a DELL 9020MT personal computer with Intel (R) Core (TM) i7mur4770 CPU @ 3.40 GHz and Windows 64-bit operating system, and the simulation software is MATLAB R2010a. To fully verify the proposed method, the calculation results of the proposed method and the reference methods are compared, and four groups of tests, which represent all of the cases in the classical and general asymptotic regimes, are carried out.

Experiment 1: the estimation results of the number of sources are compared in white Gaussian noise by two types of method; the first is the proposed method combined with the ITC methods [BIC, AIC, MDL, and Kullback information criterion (KIC)][(25)] and the second is directly using the ITC methods (BIC, AIC, MDL, and KIC). The experimental conditions are set as follows:

(1) s_1 is a binary phase shift keying (BPSK) signal with a symbol width of 10 / 31 μs and a carrier frequency of 10 MHz.

(2) s_2 is a continuous wave (CW) signal with a pulse width of 15 μs and a carrier frequency of 10 MHz.

(3) s_3 is a linear frequency modulated (LFM) signal with a pulse width of 10 + 10 · *rand*(1) μs, an initial frequency of 10 MHz, and a frequency modulation bandwidth of 10 / (1 + *rand*(1)) MHz.

(4) s_4 is a multiple phase shift keying (MPSK) signal with the Frank coding mode, the symbol width is 0.4 μs, and the carrier frequency is 50 MHz.

The number of sources is $K = 4$ and the number of antenna elements is $M = 10$, 100, 300, and 350. The mixing matrix $\boldsymbol{A}$ is generated by the random function randn. The sampling frequency is 120 MHz and the number of snapshots is $N = 300$. The observed signals are overlapped with white Gaussian noise, the SNR ranges from −10 to 30 dB, and the step size is 2 dB. Five hundred Monte Carlo simulations are carried out at each SNR and the experimental results are shown in Figs. 3(a)–3(h).

As can be seen from Figs. 3(a) and 3(b), at this time, $M/N \ll 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the classical asymptotic system. In white Gaussian noise, on the basis of the proposed method combined with the ITC methods and the direct application of the ITC methods, the estimation of the number of sources can be realized accurately at a certain SNR. The required SNR is slightly higher for the proposed method combined with the ITC methods than for other methods. In Figs. 3(c) and 3(d), M/N = 1/3, the relationship between the number of antenna elements and that of snapshots

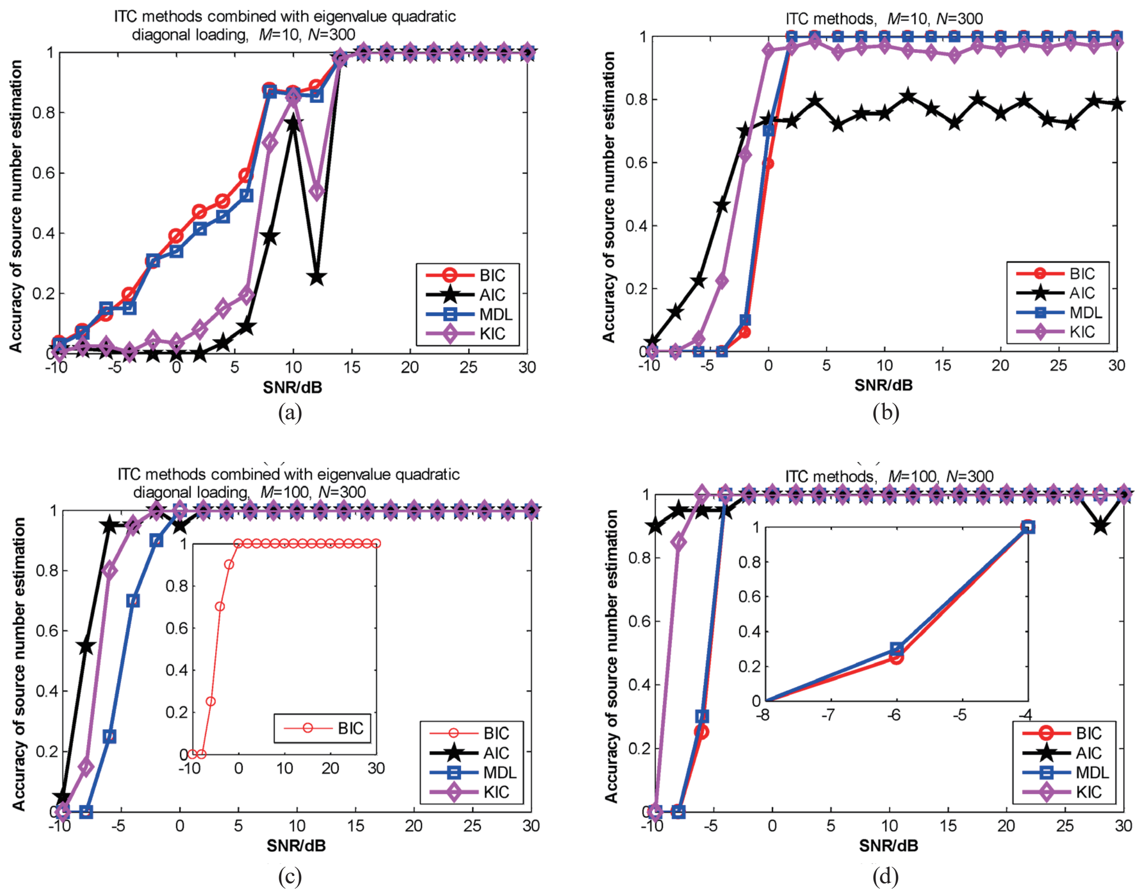
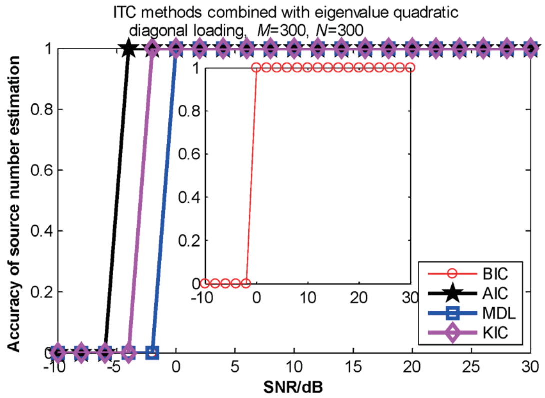
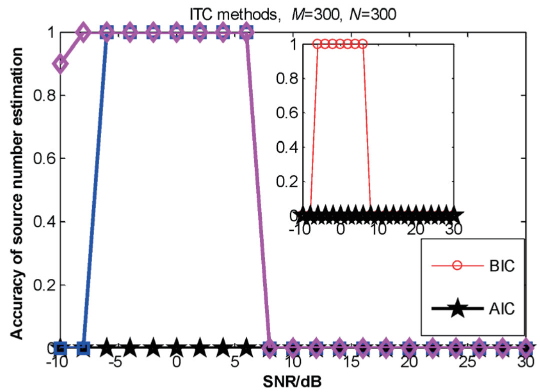


Fig. 3. (Color online) Comparison of estimation results of the number of sources between two types of method in white Gaussian noise.



(e)



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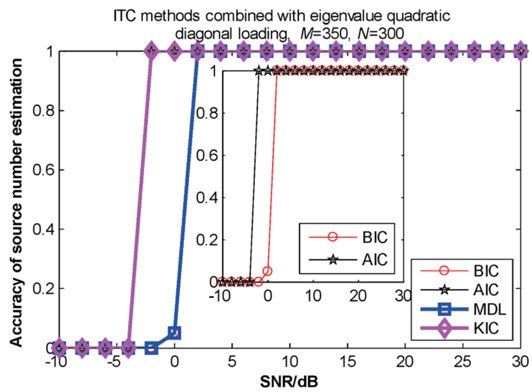

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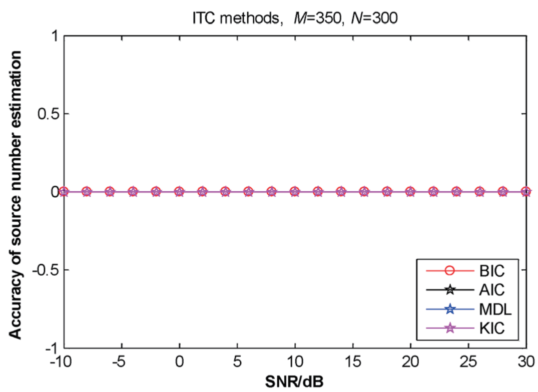


(h)

Fig. 3. (Color online) (Continued) Comparison of estimation results of the number of sources between two types of method in white Gaussian noise.

approximately meets the requirements of the classical asymptotic system. Good results can be obtained by directly using the ITC methods, and the SNRs have no clear difference. In Figs. 3(e)–3(h), $M/N \geq 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the general asymptotic regime. By the proposed method combined with the ITC methods, the accurate estimation of the number of sources can be robustly realized at a lower SNR, but the estimation fails only when using the ITC methods.

Experiment 2: the estimation results are compared in colored noise by two types of method; the first is the proposed method combined with the ITC methods (BIC, AIC, MDL, and KIC) and the second is directly using the ITC methods (BIC, AIC, MDL, and KIC). The source signals are the same as those in Experiment 1. The number of sources is K = 4, and the number of antenna elements is M = 10, 100, 300, and 350. The mixing matrix A is generated by the random function randn. The sampling frequency is 120 MHz and the number of snapshots is N = 300. The observed signals are overlapped with colored noise and the elements of its covariance matrix are expressed as $n_{ik} = \sigma_n^2 0.9^{|i-k|} \exp[(j(i-k)\pi/2)]$, $i, k = 1, 2, \cdots, M$, $j = \sqrt{-1}$. σ_n is an adjustable parameter, which is used to set the SNRs of observed signals, the variation range of SNR is

−10–40 dB, the step size is 4 dB, and 500 Monte Carlo simulations are carried out at each SNR. The experimental results are shown in Figs. 4(a)–4(g).

As can be seen from Figs. 4(a) and 4(b), $M/N \ll 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the classical asymptotic

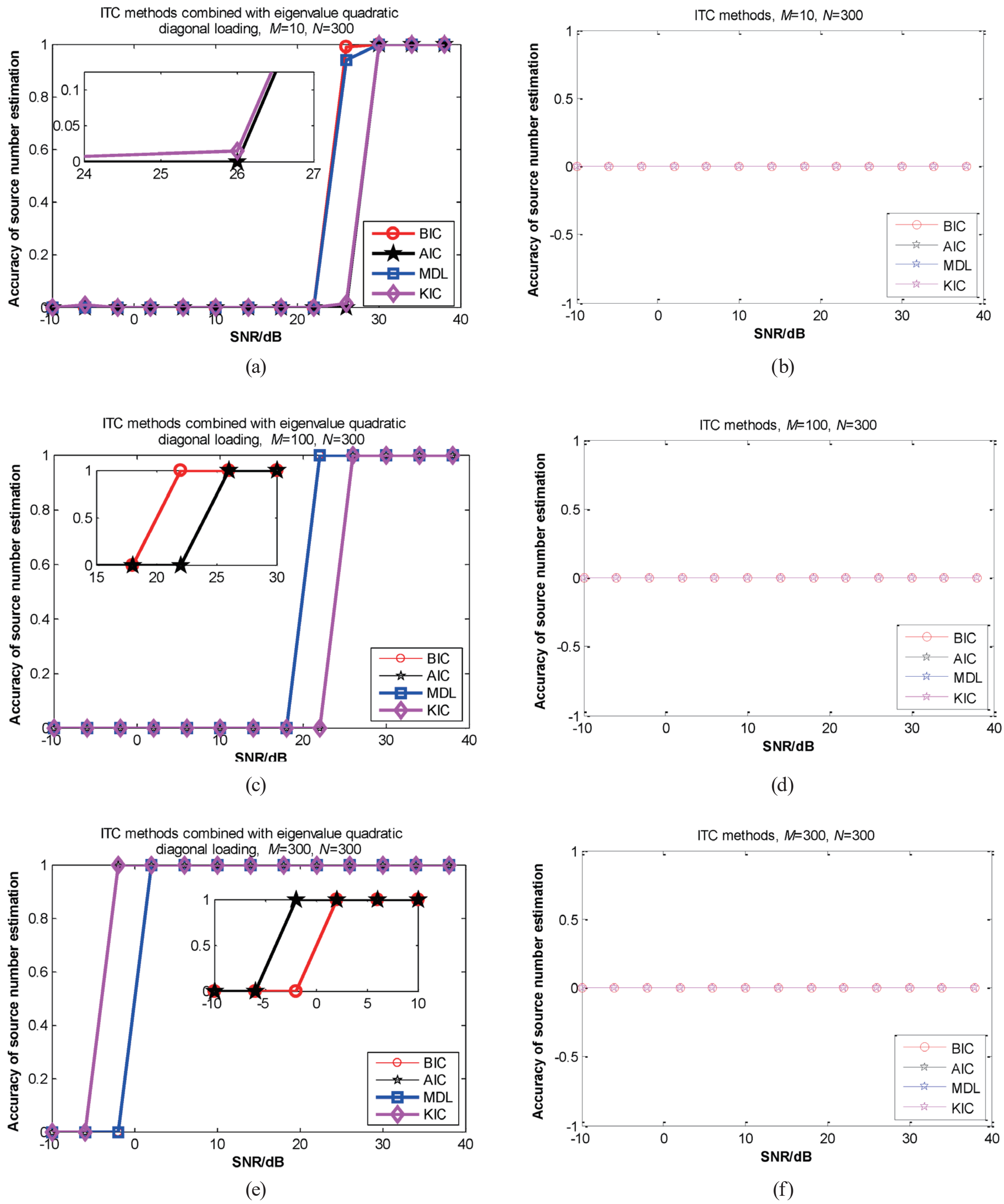
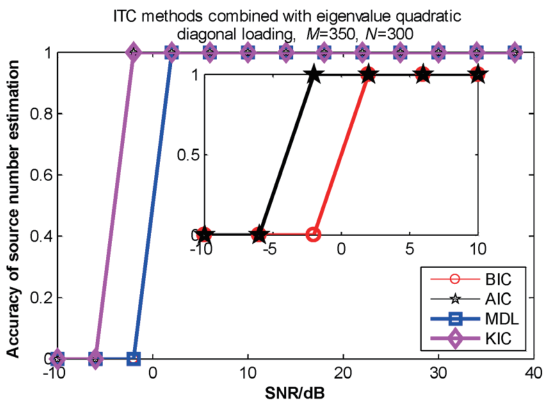


Fig. 4. (Color online) Comparison of estimation results of number of sources between the two types of method in colored noise.



(g)

Fig. 4. (Color online) (Continued) Comparison of estimation results of number of sources between the two types of method in colored noise.

system. In colored noise, the number of sources can be estimated accurately on the basis of the proposed method combined with the ITC methods at a certain SNR, but the number of sources cannot be accurately estimated without employing the proposed method. In Figs. 4(c) and 4(d), M/N = 1/3, the relationship between the number of antenna elements and that of snapshots approximately meets the requirements of the classical asymptotic system. By combining the proposed method with the ITC methods, a good effect of estimation can be achieved; otherwise, the estimation will fail. In Figs. 4(e)–4(g), $M/N \geq 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the general asymptotic regime. By the proposed method combined with the ITC methods, the accurate estimation of the number of sources can be realized at a lower SNR. However, the estimation of the number of sources will be incorrect if the proposed method is not applied.

Experiment 3: the estimation results are compared in a white Gaussian noise environment by two types of method. The first is the proposed method combined with the RMT methods [BN-AIC and Kritchman and Nadler (KN)][31] and the second is directly using the BN-AIC, KN, and the Gerschgorin circle–corrected Rao's score test–generalized Bayesian information criterion (GDE-CRSTGBIC) method.[23] The experimental conditions are set as follows:

(1) s_1 is a BPSK signal with a symbol width of 10/31 μs and a carrier frequency of 10MHz.

(2) s_2 is a CW signal with a pulse width of 15 μs and a carrier frequency of 10 MHz.

(3) s_3 is an LFM signal with a pulse width of 10 + 10 · *rand*(1) μs, an initial frequency of 10 MHz, and a frequency modulation bandwidth of 10/(1 + *rand* (1)) MHz.

(4) s_4 is a frequency shift keying (FSK) signal with a 13-bit Barker code, the symbol width is 10/13 μs, and the frequencies at the two symbols are 25 and 50 MHz.

(5) s_5 is a MPSK signal with the Frank coding mode, the symbol width is 0.4 μs, and the carrier frequency is 50 MHz.

The number of sources is K = 5 and the number of antenna elements is M = 10, 100, 300, and 350. The mixing matrix $\boldsymbol{A}$ is generated by the random function randn. The sampling frequency is 120 MHz and the number of snapshots is N = 300. The observed signals are overlapped with white Gaussian noise, the variation range of SNR is −10–30 dB, the step size is 4 dB, and 200 Monte Carlo simulations are carried out at each SNR. The experimental results are shown in Fig. 5(a)–5(f).

As can be seen from Figs. 5(a), at this time, $M/N \ll 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the classical asymptotic

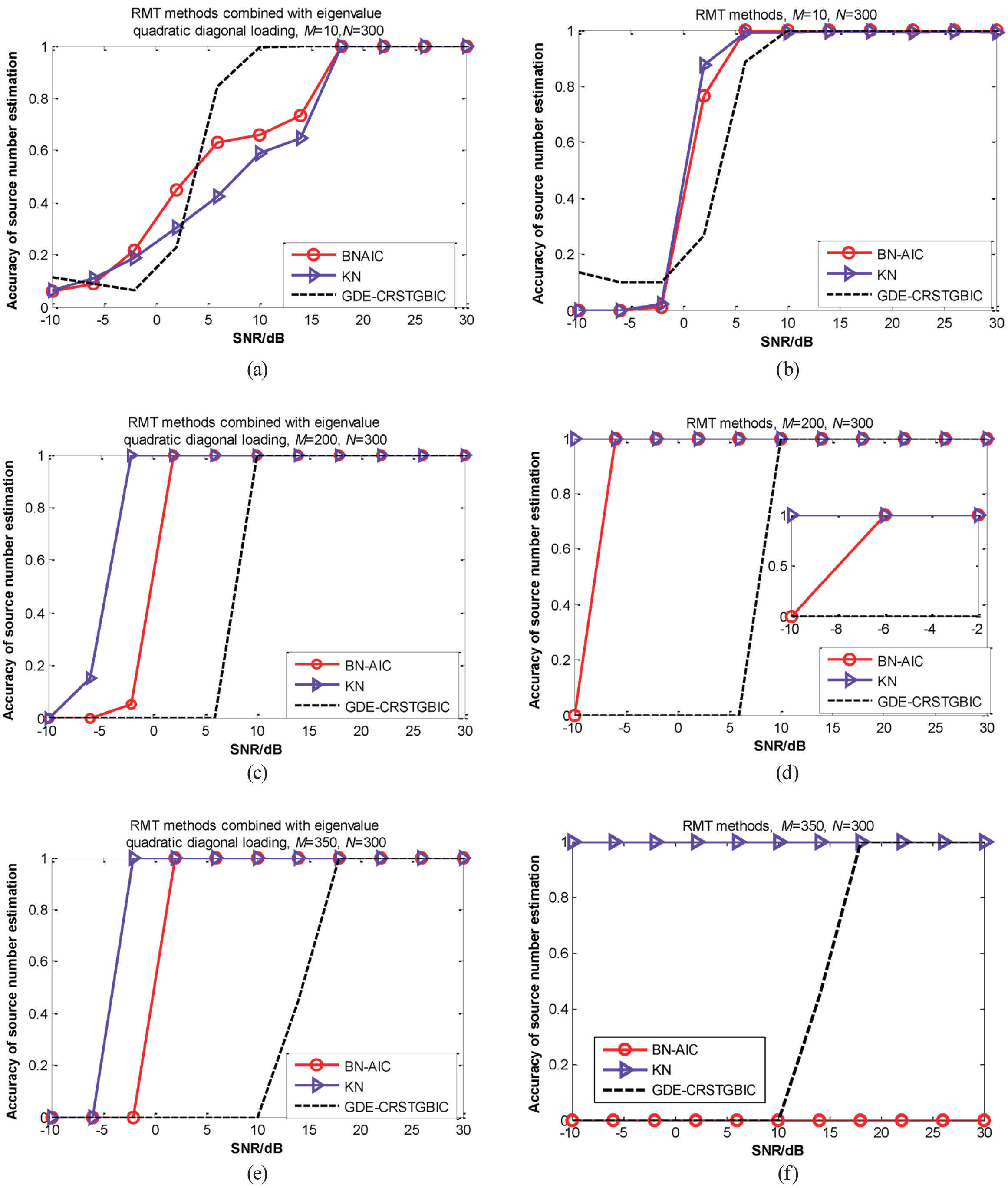


Fig. 5. (Color online) Comparison of estimation results between two types of method in white Gaussian noise.

system. In white Gaussian noise, on the basis of the proposed method combined with RMT methods, the estimation accuracy increases with SNR. When the SNR reaches 16 dB, the estimation of the number of sources can be realized with probability 1. When the SNR reaches 10 dB, the GDE-CRSTGBIC method can be used to estimate the number of sources with probability 1. Figure 5(b) shows that when the SNR reaches 6 dB, BN-AIC and KN can be used to estimate the number of sources with probability 1. In Figs. 5(c) and 5(d), M/N = 1/3, the relationship between the number of antenna elements and that of snapshots approximately meets the requirements of the classical asymptotic system. When the proposed method is combined with the BN-AIC and KN methods, their estimation accuracy increases with SNR. When the SNR increases to a certain value, the estimation accuracy can accurately estimate the number of sources with probability 1, and the requirement for the SNR of the GDE-CRSTGBIC method is lower than those of other methods. If the RMT methods are directly used, the estimation accuracies of the BN-AIC and KN methods can reach probability 1 when the SNR reaches a certain value. In Figs. 5(e) and 5(f), $M/N \geq 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the general asymptotic regime. By combining the proposed method with the BN-AIC and KN methods, they can accurately estimate the number of sources with probability 1 when the SNR reaches a certain value. By only using the BN-AIC and KN methods, the KN method can be used to accurately estimate the number of sources with probability 1 when the SNR reaches a certain value, but the BN-AIC method failed when the number of antenna elements is more than that of snapshots.

Experiment 4: the estimation results are compared in a colored noise environment by two types of method; the first is the proposed method combined with the RMT methods (BN-AIC and KN) and the second is directly using the RMT methods (BN-AIC and KN) and the GDE-CRSTGBIC method. The source signals are the same as those in Experiment 3.

The number of sources is K = 5 and the number of antenna elements is M = 10, 100, 300, and 350. The mixing matrix $\boldsymbol{A}$ is generated by the random function randn. The sampling frequency is 120 MHz and the number of snapshots is N = 300. The observed signals are overlapped with colored noise, and the elements of its covariance matrix are expressed as $n_{ik} = \sigma_n^2 0.9^{|i-k|} \exp[(j(i-k)\pi/2)]$, $i,k = 1, 2, \cdots, M$. σ_n is an adjustable parameter, which is used to set the SNRs of observed signals, the variation range of SNR is −10–30 dB , the step size is 4 dB, and 200 Monte Carlo simulations are carried out at each SNR. The experimental results are shown in Figs. 6(a)–6(f).

As can be seen from Figs. 6(a) and 6(b), at this time, $M/N \ll 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the classical asymptotic system. In colored noise, on the basis of the proposed method combined with RMT methods, the number of sources can be estimated accurately at a certain SNR. If the proposed method is not applied, only the GDE-CRSTGBIC method can be used to correctly estimate the number of sources, and the SNR required by this method to achieve the accuracy of probability 1 is slightly lower than those of the BN-AIC and KN methods based on the proposed method. In Figs. 6(c) and 6(d), M/N = 1/3, the relationship between the number of antenna elements and that of snapshots approximately meets the requirements of the classical asymptotic system. By combining the proposed method with RMT methods, the estimation accuracy of the BN-AIC

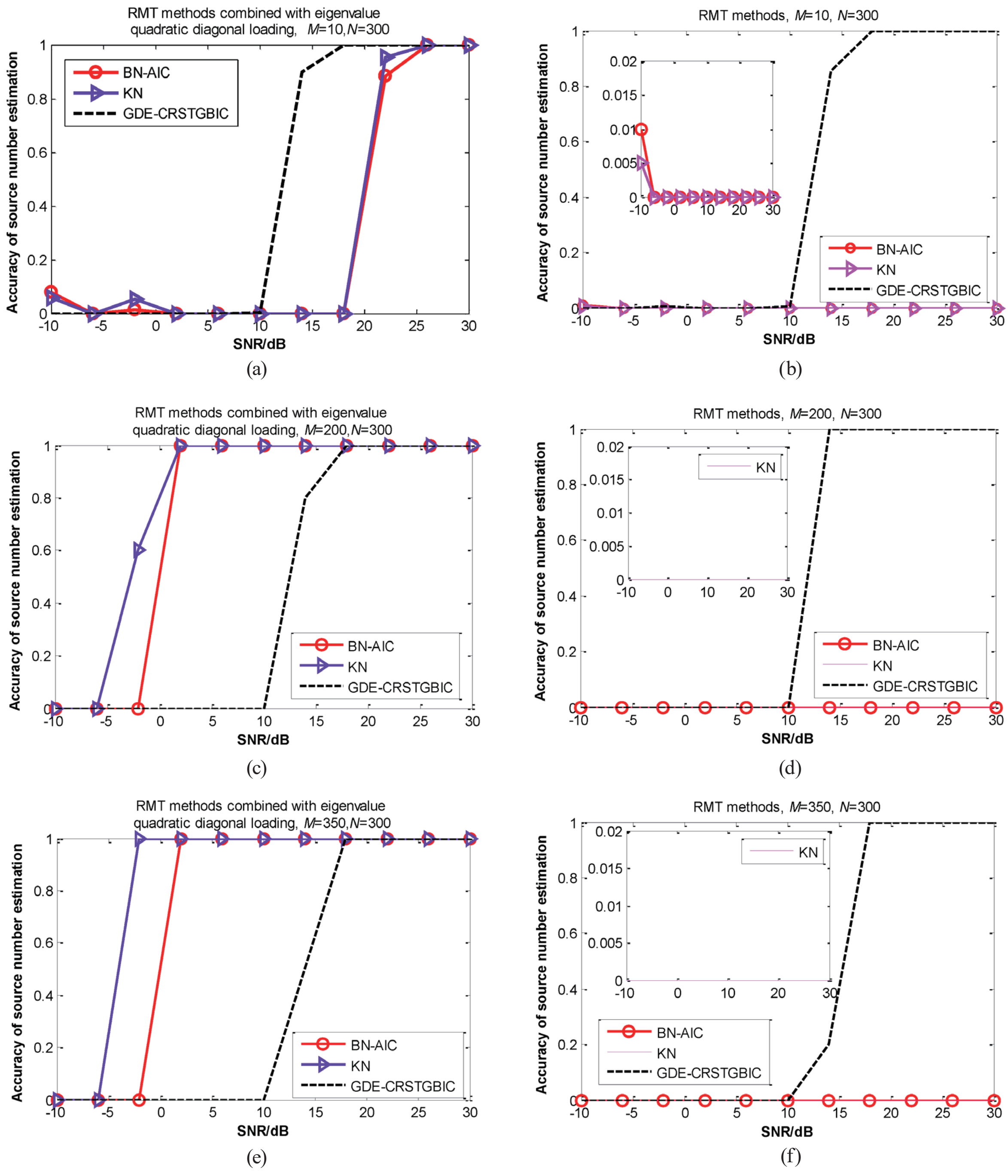


Fig. 6. (Color online) Comparison of estimation results between the two types of method in colored noise.

and KN methods is improved with increasing SNR. When the SNR reaches a certain value, the number of sources can be estimated accurately with probability 1. The number of sources cannot be estimated accurately using only the BN-AIC and KN methods. In Figs. 6(e) and (f), $M/N \geq 1$, the relationship between the number of antenna elements and that of snapshots meets the requirements of the general asymptotic regime. By combining our proposed method and RMT

Table 1
Advantages and disadvantages of our proposed method with various existing methods.

Methods	Advantages	Disadvantages
ITC methods (BIC, AIC, MDL, KIC)	Applicable to: • Classical asymptotic regime • White Gaussian noise • Can get good results at lower SNR	Not applicable to: • General asymptotic regime • Colored noise
Eigenvalue quadratic diagonal loading combined with ITC methods (BIC, AIC, MDL, KIC)	Applicable to: • Classical asymptotic regime • General asymptotic regime • White Gaussian noise • Colored noise	
RMT methods (BN-AIC, KN)	Applicable to: • Classical asymptotic regime • General asymptotic regime • White Gaussian noise	Not applicable to: • Colored noise
GDE-CRSTGBIC method	Applicable to: • Classical asymptotic regime • General asymptotic regime • White Gaussian noise • Colored noise	•Need higher SNR
Eigenvalue quadratic diagonal loading combined with BN-AIC or KN	Applicable to: • Classical asymptotic regime • General asymptotic regime • White Gaussian noise • Colored noise	

methods, the estimation accuracy of the BN-AIC and KN methods is improved with increasing SNR. When the SNR reaches a certain value, the number of sources can be estimated accurately with probability 1 and with a smaller SNR than GDE-CRSTGBIC. Without the proposed method, but only using the BN-AIC and KN methods, the estimation failed.

From the previous experiments, the advantages and disadvantages of combining of our proposed method with various existing methods are shown in Table 1.

6. Conclusions

The estimation method of the number of sources proposed in this paper belongs to an important area of antenna array signal processing in the field of sensor technology. In an actual environment, when an antenna array is used for signal reception, the proportional relationship between the number of antenna elements and that of signal snapshots and whether the noise of observed signals is mixed with white Gaussian or colored noise is unknown. In this study, an eigenvalue quadratic diagonal loading method for estimating the number of sources is designed. By combining the proposed method with the existing ITC methods, the ITC methods are extended to general asymptotic regimes, and the noise environment can be white Gaussian or colored noise. At the same time, this method extends the applicable scope of the RMT methods, which can be applied to the environment of colored noise. Compared with the existing eigenvalue diagonal loading methods, the proposed method adopts the eigenvalue secondary diagonal loading, which is equivalent to the second correction of the eigenvalue distribution, and makes the estimation more robust. At the same time, the quadratic diagonal loading of the eigenvalues of the covariance matrix, combined with the ITC or RMT methods, does not affect the function for estimating the number of sources in a white Gaussian noise environment.

In the follow-up research on the problem of estimating the number of sources, we will continue to conduct in-depth research on other practical problems, mainly including (1) the estimation of the number of sources and DOA in the case of non-Gaussian distribution observed data and (2) the estimation of the number of sources and DOA when the received signals of each antenna element do not meet independent conditions.

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About the Authors

Chuanchuan Wang received his B.A., M.S., and Ph.D. degrees from Shijiazhuang Mechanical Engineering College, China, in 2007, 2009, and 2013, respectively. Since 2014, he has been a research assistant at the State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System. He is the author of more than 50 articles and co-author of 23 inventions and one book. His research interests include blind signal processing and effectiveness evaluation theory.
(wangchuan1083@126.com)

Yonghu Zeng received his B.S., M.S., and Ph.D. degrees from the National University of Defense Technology, China, in 1994, 1997, and 2004, respectively. Since 2012, he has been a researcher at the State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System. He has co-authored more than 80 articles, 6 books, and 26 inventions. His research interests include radar signal processing and effectiveness evaluation theory. (duoduo17wang@163.com)

Zhipeng Li received his B.S. and M.S. degrees from the University of Aerospace Engineering in 2006 and 2008, respectively. Since 2013, he has been a research assistant at the State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System. He has co-authored more than 10 articles and 5 inventions. His research interests include signal processing and effectiveness evaluation theory.
(471680179@qq.com)

Liandong Wang received his B.S., M.S., and Ph.D. degrees from the National University of Defense Technology, China, in 1989, 1993, and 2000, respectively. Since 2012, he has been a researcher at the State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System. He has co-authored more than 100 articles, 10 books, and 30 inventions. His research interests include signal processing, electromagnetic environment effects, and effectiveness evaluation theory. (874353112@qq.com)