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Latest Progress and Challenges in Multi-frequency and Multi-global Navigation Satellite System Precise Point Positioning Ambiguity Resolution Technology

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The development of multi-frequency Global Navigation Satellite System (GNSS) signals and multi-GNSS constellations and precise positioning sensors has significantly improved positioning performance. As a representative of highly precise positioning technology, precise point positioning ambiguity resolution (PPP AR) technology has also made significant progress. In this paper, the latest progress and challenges of multi-frequency and multi-GNSS PPP AR technology are comprehensively discussed. According to the time development sequence of multi-frequency and multi-GNSS PPP AR, we present the development of multi-GNSS dual-frequency PPP AR, multi-frequency single-system PPP AR, and multi-frequency and multi-GNSS PPP AR in turn and explain the problems with key technologies that needed to be solved in the process of realizing multi-frequency and multi-GNSS PPP AR, such as mathematical models, ambiguity resolution algorithms, and inter-system and inter-frequency biases. Further research challenges of PPP AR popularization and application are also summarized.

1. Introduction

The development of the Global Navigation Satellite System (GNSS) and corresponding positioning sensors has brought revolutionary changes to high-precision positioning technology. As the representative of positioning technology, precise point positioning (PPP) is used in many engineering applications and scientific research fields, such as high-precision static and kinematic positioning, precise timing, precise orbit determination of low-orbit satellites, tidal and sea level monitoring, water vapor inversion and ionospheric monitoring, seismic monitoring, and earth plate movement monitoring, because of its centimeter-level positioning, no limits on operating distance, flexible operation, and many other significant advantages.(1–8) It has

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important applications in both research and development. Therefore, in recent years, the major GNSS, including Global Positioning System (GPS), GLObal Navigation Satellite System (GLONASS), BeiDou Navigation Satellite System (BDS), and Galileo satellite navigation system (Galileo) have deployed PPP services transmitted from satellite systems. The BDS-3 has taken the lead in launching PPP services via the B2b signal in August 2020, which indicates that PPP technology has been further developed and is expected to become an important technical means of high-precision navigation, positioning, and timing.(9,10)

However, traditional PPP needs about 30 minutes to finish initialization for the first time, which limits its application in traditional high-precision positioning fields such as deformation monitoring, as well as emerging high-precision positioning fields such as automatic (assisted) driving, unmanned aerial vehicles (UAVs), and wearable devices.(11) The main reason for the long first initialization time of PPP is that the PPP observation equation contains too many parameters that must be determined, such as the three-dimensional position of the receiver, the receiver clock offset, the atmospheric delay errors, and the ambiguity, which enhances the correlation between the parameters sought,. Therefore, it takes a long time to accumulate enough effective observation information to accurately separate various parameters. Over the years many scholars have devoted their time to solving the problem of how to shorten the first initialization time of PPP. (11)

Since PPP ambiguity resolution (PPP AR) technology was proposed, its accuracy in terms of positioning and reliability has been improved, and the first initialization time has been shortened.(12–14) Over the years, although the technology has been improved and perfected by many scholars, the first initialization time of the single-GNSS PPP AR is still about 20 min, which remains the main bottleneck restricting its wider application.(15–21)

In recent years, the modernization of GPS, the revival of the GLONASS system, the successive completion of BDS-3 and the Galileo system, and the successive completion of the Indian regional navigation satellite system (IRNSS) and the Japanese quasi-zenith satellite system (QZSS) have brought the development of GNSS to a new stage. At the same time, this also brings new opportunities for research on rapid PPP AR. Compared with a single GPS observation, multi-system GNSS has richer observational information that can enhance the spatial geometric distribution among satellite receivers and increase the additional observations made by surveying adjustment systems, all of which are conducive to the improvement of positioning performance.(22) In addition, GPS Block IIF satellites and all BDS-3 and Galileo satellites can transmit signals at three frequencies or more. It has been established in relative positioning that, compared with dual-frequency observations, the use of three-frequency signals significantly improves the ambiguity search space, improves the efficiency of ambiguity resolution, and speeds up the resolution of ambiguities.(23–25) This also undoubtedly indicates a possible route to improve the initialization time of PPP AR.

Based on these points, making full use of multi-frequency and multi-GNSS observations is one of the ways to solve the time demands of PPP AR. Scholars also have carried out many studies on this issue, and in this paper, we summarize the results systematically. In fact, multi-frequency and multi-GNSS PPP are two parallel research routes, but the research on multi-GNSS PPP AR was carried out prior to that on multi-frequency PPP AR. Therefore, research on multi-GNSS and dual-frequency PPP AR is discussed first in this paper; then the progress on multi-frequency and single-GNSS PPP AR is summarized. The research progress on multi-frequency and multi-GNSS PPP AR is then presented, and finally, the challenges faced by multi-frequency and multi-GNSS PPP AR are pointed out and conclusions are drawn.

2. Progress in Multi-GNSS PPP AR

Early research on multi-GNSS and dual-frequency PPP AR mainly focused on the combination of GPS and GLONASS. Compared with GPS satellites, GLONASS satellite adopts frequency division multiple access (FDMA) technology, so an inter-frequency bias exists in both the pseudo-range and phase observations of different satellites at different frequencies. Although the function model of phase inter-frequency bias can be modeled, the pseudo-range inter-frequency bias is highly correlated with the type of receiver and antenna and the version of receiver firmware, which is difficult to model. Therefore, GLONASS PPP ambiguity is difficult to resolve.(26)

Jokinen *et al.* used GLONASS observations to assist GPS PPP AR.(27) In this strategy, only the GPS satellite ambiguity was fixed, whereas GLONASS satellite ambiguity was set as a float ambiguity. The results showed that, compared with the single GPS PPP AR, the first initialization time of GPS PPP AR can be improved by 5% with GLONASS observations.(27) Liu *et al.* estimated the uncalibrated phase delay (UPD) on GLONASS satellites using stations with the same types of receiver and implemented GLONASS PPP AR based on the UPD product.(28) The results showed that, after 2 h of PPP data processing, the positioning accuracies of PPP float solutions were 0.66, 1.42, and 1.55 cm in E, N, and U directions, whereas the PPP AR could improve the positioning accuracies to 0.38, 0.39, and 1.39 cm in the three directions, respectively.(28) Geng and Bock proposed the application of external ionospheric products to GLONASS PPP AR to assist different types of receivers in estimating the UPD of GLONASS satellites.(29) This method can significantly attenuate the influence of pseudo-range inter-frequency bias of different receivers on UPD estimations. When there is no external ionospheric product constraint, the proportion of satellite wide-lane UPD within 0.15 cycles increases to 51.7%, and when an external ionospheric product constraint is applied, the proportion of satellite wide-lane UPD within 0.15 cycles increases to 92.4%.(29) Liu *et al.* realized the GPS+GLONASS dual-system PPP AR, and the results showed that the fixed rates of the single GPS PPP AR at 5 min and 10 min were 11.7 and 46.8%, respectively.(30) However, the fixed rates of the GPS/GLONASS dual-system PPP AR were increased to 73.71 and 95.83% at 5 and 10 min, respectively.(30)

Because the BDS-2 provides navigation and positioning services in the Asia-Pacific region, the BDS/GPS dual-system PPP AR is of considerable research interest. However, different from GPS observations, a systematic bias that is related to elevation in the BDS pseudo-range observations arises, which changes with the elevation; the maximum bias does not exceed 1 m. This bias is mainly derived from the BDS satellite pseudo-range multi-path.(31,32) Pseudo-range multipath errors of a BDS satellite seriously affect the MW observations and hence the wide-lane UPD estimations and wide-lane ambiguity accuracy, so they must be corrected before realizing BDS PPP AR.(33) Wanninger and Beer established the pseudo-range multipath empirical model for Beidou MEO and IGSO satellites and verified the correctness of the model using single-frequency PPP.(33) On the basis of the empirical model, Li analyzed the influence of the BDS pseudo-range multipath on BDS UPD estimations.(34) The results showed that the average utilization of wide-lane ambiguity could be improved from 80.4 to 91.8% after taking this error into account. At the same time, BDS PPP AR was implemented by BDS UPD with a corrected pseudo-range multi-path. Compared with the BDS float PPP, the ambiguity-fixed PPP improved the positioning accuracy in the E, N, and U directions by 13.2, 2.7, and 28.4%, respectively.(34) However, owing to the lack of a pseudo-range multipath model for the GEO satellite, this experiment only fixed the BDS, MEO, and IGSO satellite ambiguities but not those of the GEO satellite. Li *et al.* weakened the pseudo-range multipath of the BDS GEO satellite by extracting the low-frequency component and correcting the pseudo-range observation value through wavelet transform and realized BDS PPP AR with the GEO/IGSO/MEO satellites.(35) The experimental results showed that, compared with the BDS PPP AR that only fixed the IGSO and MEO satellites, the addition of GEO satellites could significantly improve the visible number of BDS satellites, optimize the spatial geometry configuration, shorten the first initialization time of BDS PPP AR, and improve the positioning accuracy.(35) Li *et al.* implemented the GPS/BDS dual-system PPP AR by using the adaptively partial ambiguity resolution method and compared the positioning performance of the BDS single-system PPP AR, GPS single-system PPP AR, and GPS/BDS dual-system PPP AR in a large number of experiments. The results showed that the first initialization times of a GPS single-system PPP AR in static and dynamic positioning mode were 21.7 and 33.6 min, whereas the first fixed times of the GPS/BDS dual-system PPP AR in the two modes were significantly shortened to 16.9 and 24.6 min, respectively.(36) Liu *et al.* realized the GPS/BDS PPP AR using the whole constellation of BDS satellites and analyzed the influence of GEO satellites on AR.(37) The conclusion pointed out that, in the small area observation network, the GEO orbit error can be absorbed by the UPD product and the combination of GEO satellites can improve the ambiguity fixed rate.(37)

At the same time the BDS satellite was launched, Galileo was also being constructed in a manner that provided conditions for the realization of PPP AR by combining the observations of GPS, GLONASS, BDS, and Galileo. Li *et al.* proposed the orbit determination, clock estimation, and positioning model of GPS/GLONASS/BDS/Galileo, and compared the positioning performance of a GPS single-system PPP with the four-system PPP.(38,39) The results showed that the PDOP of the single GPS system varied from 2 to 6, whereas the PDOP of the four-system was stable at about 1.5. Compared with the GPS single-system PPP, the GPS/GLONASS/BDS/Galileo four-system PPP shortened the convergence time by 70% and improved the positioning accuracy by about 25%. In addition, when the elevation is 30° or 40°, the positioning performance of the GPS single-system PPP is significantly decreased, and the positioning results are unavailable 30 or 60% of the time, respectively. In contrast, the four-system PPP still maintains good positioning performance.(38,39) To avoid amplifying measurement noise when multiple system observations are combined, Lou *et al.* implemented single-frequency and dual-frequency PPP of GPS/GLNAOSS/BDS/Galileo based on the undifferenced and uncombined model.(40) The results showed that the convergence time of dual-frequency GPS/GLNAOSS/

BDS/Galileo PPP was improved by about 60% compared with that of the GPS single-system dual-frequency PPP; compared with the GPS single-system single-frequency PPP, the positioning accuracy of the four-system single-frequency PPP was improved by about 25%.(40) Liu *et al.* proposed the undifferenced and uncombined PPP model of GPS/GLNAOSS/BDS/Galileo system taking into account both GLONASS inter-frequency bias and inter-system bias and analyzed the characteristics of inter-frequency bias and inter-system bias.(41) Li *et al.* realized the GPS/GLNAOSS/BDS/Galileo four-system UPD estimation and resolved the GPS/GLNAOSS/BDS/Galileo four-system PPP ambiguity.(42) The results showed that the first initialization time of the four-system PPP AR was the fastest and the positioning accuracy was the highest. The first initialization times of the GPS single-system PPP AR, GPS/GLONASS dual-system PPP AR, GPS/BDS dual-system PPP AR, and GPS/Galileo dual-system PPP AR were 18.07, 12.10, 13.21, and 15.36 min, respectively, whereas the first initialization time of the four-system PPP AR was increased to 9.21 min.(42)

To clearly demonstrate the performance of multi-GNSS PPP AR, the first initialization time and positioning accuracy of PPP AR with different GNSS combinations are shown in Fig. 1 and Table 1, respectively. Figure 1 shows the first initialization times of GPS PPP AR, GPS/GLONASS PPP AR, GPS/GLONASS/BDS-2 PPP AR, and GPS/GLONASS/BDS-2/Galileo PPP AR. The first initialization time of the four-system PPP AR was the shortest. Table 1 shows the positioning accuracies of GPS PPP AR, GPS/GLONASS PPP AR, GPS/GLONASS/BDS-2 PPP AR, and GPS/GLONASS/BDS-2/Galileo PPP AR in E, N, and U directions after the undifferenced ambiguity was fixed. According to the statistical results, the positioning accuracy of the four-system PPP AR was the highest. Multi-GNSS constellations considerably increase

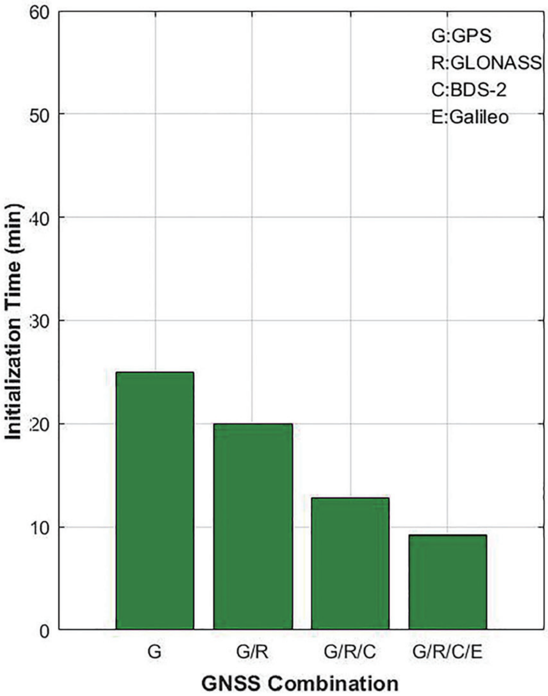

Fig. 1. (Color online) First initialization time of multi-GNSS PPP AR.

Table 1
Positioning bias in E, N, and U directions after first initialization of multi-GNSS PPP AR.

GNSS Combination	Position Bias (unit: cm)		
	E	N	U
GPS	2.5	2.3	2.7
GPS/GLONASS	2.0	1.9	2.2
GPS/GLONASS/BDS	1.2	1.1	1.8
GPS/GLONASS/BDS/Galileo	0.9	0.8	1.5

the number of satellites that can be used for positioning and further enhance the spatial geometry structure between satellite and receiver. Therefore, the positioning performance of multi-GNSS PPP AR has been significantly improved, especially in terms of the first initialization time.

3. Progress in Multi-frequency PPP AR

Compared with the research on multi-GNSS PPP AR, the research on multi-frequency PPP AR started relatively late, mainly because of the late launch of satellites capable of broadcasting multi-frequency signals. With the successive launch of GPS Block IIF satellites capable of broadcasting triple-frequency signals and the completion of the BDS-2, the research era of multi-frequency PPP AR began.

However, before the actual implementation of multi-frequency PPP AR, scholars found some bias among the different frequency signal observations. For example, after the launch of a few GPS Block IIF satellites, Montenbruck *et al.* found that there was bias between the phase observations of the ionosphere-free combination at the L1/L2 frequency and the phase observations of the ionosphere-free combination at the L1/L5 frequency, which could reach a maximum of 10 cm.(43) Montenbruck *et al.* later pointed out that the bias in the periodic variation was mainly due to the periodic variation between the orbital plane of the satellite and the elevation of the earth and the sun, which led to a variation of the internal hardware temperature of the satellite, causing a periodic variation of the satellite clock frequency. This periodic bias is called the inter-frequency clock bias (IFCB).(44) Li *et al.* proposed an estimation method for satellite IFCB and modeled it using a fourth-order spherical harmonic function, pointing out that the accuracy in the of satellite IFCB over a five day period was better than 7 cm.(45,46)

As the number of GPS Block IIF satellites increased, some scholars began to study multi-frequency GPS PPP AR. Tegedor and Øvstedal proposed a triple-frequency GPS PPP function and stochastic model of the ionosphere-free combination at L1/L2 frequency and L1/L5 frequency, and pointed out that IFCB should be taken into account when dealing with triple-frequency PPP.(47) Different from the PPP model of the ionosphere-free combination at L1/L2 frequency and L1/L5 frequency, Elsobeiey *et al.* proposed a PPP model with the combination of triple-frequency observations, verified different combination coefficients through analysis and experiments, obtained the optimal triple-frequency combination coefficient, and realized GPS triple-frequency PPP based on the combination coefficient.(48) The results showed that, compared with the dual-frequency GPS PPP, the positioning accuracy and the initialization time of triple-frequency GPS PPP can be improved by about 10%.(48)

With the increasing number of GPS Block IIF satellites, some scholars began to invest in the research of multi-frequency GPS PPP AR. Liu *et al.* analyzed satellite code bias (SCB) and IFCB in the positioning performance of GPS triple-frequency PPP AR.(49) Experimental results showed that SCB mainly affected the first initialization time of the GPS triple-frequency PPP AR and had a greater impact on the kinematic PPP AR. Compared with the kinematic PPP AR without SCB correction, the first initialization time of the kinematic PPP AR with SCB correction was shortened by about 29%. However, SCB had no significant impact on the positioning accuracy of PPP AR. IFCB can affect both the first initialization time and positioning accuracy of PPP AR. Compared with the PPP AR without IFCB correction, the first initialization time of the PPP AR with IFCB correction was shortened by about 64.3 %, and the positioning accuracy was increased by about 40 to 60%.(49)

In AR algorithm research, Geng and Bock presented a triple-frequency ionosphere-free combination PPP AR algorithm, an algorithm first using the HMW combination calculating extra-wide-lane ambiguity, then using the fixed extra-wide-lane ambiguity to assist wide-lane ambiguity resolution, and finally using the fixed wide-lane ambiguity to assist narrow-lane ambiguity resolution.(50) The proposed method was verified using the simulated GPS triple-frequency observation data. The results showed that 99% of the narrow-lane ambiguity was fixed within 65 s using the triple-frequency observation data, while only 64% of the narrow-lane ambiguity was successfully fixed within 150 s using the dual-frequency observation data.(50) With the completion of the BDS-2, Guo *et al.* evaluated the positioning performances of three BDS triple-frequency PPP models, including two ionosphere-free combination models, a triple-frequency ionosphere-free combination model, and a triple-frequency uncombined model.(50) The results showed that the positioning performances of the three models were comparable. Adding additional frequency observations did not significantly improve the positioning performance of static PPP. In dynamic PPP, when the quality of the B1/B2 frequency observations was poor, adding the third frequency observation was of great significance in improving the positioning performance.(51) Gu *et al.* realized BDS-2 triple-frequency PPP AR based on an undifferenced and uncombined model.(52) Experiments showed that the triple-frequency PPP AR significantly improved the first initialization time and positioning accuracy. However, it was still difficult to rapidly fix the BDS-2 L1 ambiguity.(52) Li *et al.* proposed a method of multi-frequency original-frequency UPD estimation and multi-frequency PPP AR based on an undifferenced and uncombined model and used this method to evaluate the positioning performance of BDS-2 triple-frequency float PPP, BDS-2 dual-frequency PPP AR, and BDS-2 triple-frequency PPP AR.(53) The results showed that, compared with the triple-frequency float PPP, the triple-frequency PPP AR significantly improved the positioning accuracy and shortened the first initialization time. Among the three groups of solutions, the positioning performance of the triple-frequency PPP AR was the best. Compared with the dual-frequency PPP AR, the positioning accuracy of the triple-frequency PPP AR in the E, N, and U directions was improved by 16.6, 10.0, and 11.1%, respectively, and the first initialization time was decreased by 10%. Limited by the number of BDS-2 visible satellites, the accuracies of precise orbit and clock, and the accuracy of PCO/PCV error correction, the contribution of additional frequency observations to the PPP AR has not been fully demonstrated; this issue merits further research and discussion.(53) With the completion of BDS-3, Li *et al.* evaluated the BDS-3 five-frequency PPP AR.(54) The results showed that the first initialization time of the BDS-3 five-frequency PPP AR was 25.4% faster than that of the dual-frequency PPP AR.(54)

To further clarify the contribution of GNSS multi-frequency signals to PPP AR, we compared the positioning performances of dual-frequency PPP AR and triple-frequency PPP AR. Figure 2 shows the first initialization time of GPS, Galileo, and BDS-3 dual-frequency and triple-frequency PPP AR. As can be seen from the figure, the first initialization time of GNSS PPP AR in all systems was improved after adopting the triple-frequency signals, shortening the time by about 15%. Table 2 shows the positioning bias in E, N, and U directions of GPS, Galileo, and BDS-3 dual-frequency and triple-frequency PPP AR after the ambiguity was fixed. As can be seen from the table, compared with dual-frequency PPP AR, triple-frequency PPP AR had higher positioning accuracy. The positioning performance of PPP AR could be improved after using multi-frequency GNSS signals, mainly because multi-frequency data can generate more linear combinations and richer observational information, improve the ambiguity search space, and speed up the efficiency of the ambiguity search.

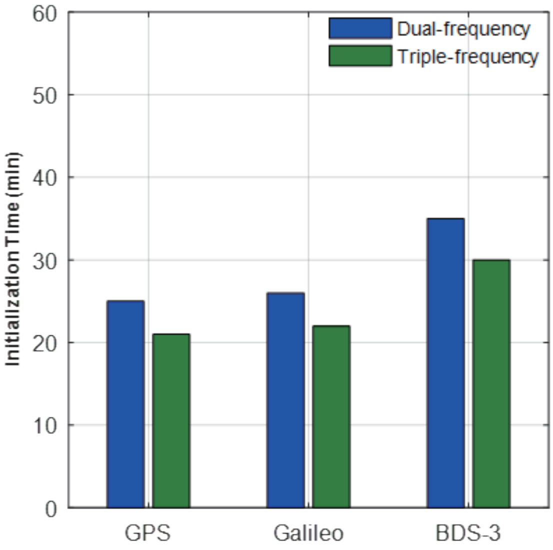

Fig. 2. (Color online) First initialization time of multi-GNSS PPP AR.

Table 2
Positioning bias in E, N, and U directions of dual-frequency and triple-frequency PPP AR

GNSS combination	Frequency number	Position Bias (unit: cm)		
		E	N	U
GPS	Dual	2.5	2.3	2.7
	Triple	2.1	1.9	2.5
Galileo	Dual	2.6	2.2	2.8
	Triple	2.2	2.0	2.5
BDS-3	Dual	2.8	2.7	3.0
	Triple	2.5	2.2	2.6

4. Progress in Multi-GNSS and Multi-frequency PPP AR

With the improvement in GNSS, as well as the study of various biases in multi-GNSS and multi-frequency observations, many scholars have successfully realized multi-frequency and multi-GNSS PPP AR. Li *et al.* implemented PPP AR based on an undifferenced and uncombined model by using the GPS/Galileo/BDS triple-system and triple-frequency observation data.(55) The results showed that the average 3D positioning accuracy of the kinematic PPP AR was 2.2 cm, and the average initialization time was 10.8 min, which was shortened by 15.6% compared with that of dual-frequency PPP AR.(55)

Using an ionospheric-free model, Geng *et al.* proposed a PPP multi-frequency and multi-GNSS single epoch wide-lane ambiguity resolution method.(56) Different from selecting reference stars in each GNSS system in turn to form the single-differenced ambiguities, this method selected only one reference star within each GNSS system to generate the single-differenced wide-lane ambiguities by correcting the phase bias estimated in advance to achieve multi-frequency and multi-GNSS wide-lane PPP AR. The proposed method was verified using the GPS/BDS/Galileo/QZSS triple-frequency and four-system observational data. The results showed that a single-epoch wide-lane PPP AR could be realized based on 91.2% of the experimental results of IGS stations. The single-epoch wide-lane PPP AR could be realized based on 99.31% of the experimental results of vehicle dynamic positioning. The single-epoch positioning accuracies of E, N, and U directions were within 0.29, 0.35, and 0.77 m, respectively.(56)

Figure 3 and Table 3 further show the positioning performance of multi-frequency and multi-GNSS of PPP AR. Figure 3 shows the first initialization time of dual-frequency and triple-frequency PPP AR under different GNSS combinations. The more frequencies and satellites used for PPP AR, the shorter the first initialization time. Table 3 shows the positioning bias of dual-frequency and triple-frequency PPP AR with different GNSS combinations. The more

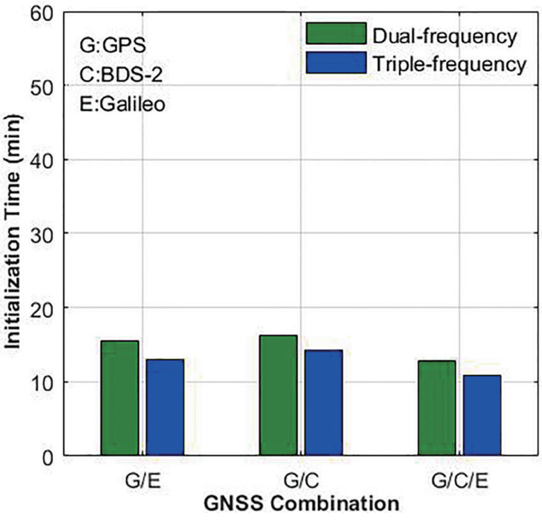

Fig. 3. (Color online) First initialization time of multi-frequency and multi-GNSS PPP AR.

Table 3
Positioning bias in E, N, and U directions of dual-frequency and triple-frequency PPP AR with different GNSS combinations.

GNSS combination	Frequency number	Position Bias (unit: cm)		
		E	N	U
GPS/Galileo	Dual	1.5	1.4	2.0
	Triple	1.3	1.1	1.9
BDS/Galileo	Dual	1.7	1.5	2.2
	Triple	1.4	1.3	2.0
GPS/BDS/Galileo	Dual	1.1	1.0	1.8
	Triple	1.0	0.9	1.5

frequencies and satellites used for PPP AR, the higher its positioning accuracy. In general, the combination of multi-frequency and multi-GNSS plays a significant role in improving PPP AR positioning performance.

5. Conclusions

The development of multi-frequency GNSS signals and multi-GNSS constellations as well as the corresponding precise positioning sensors improves the first initialization time and the positioning accuracy of PPP AR. However, many scholars have conducted systematic and comprehensive studies on multi-frequency and multi-GNSS PPP AR models, methods, and error processing, and the area still faces many problems and challenges worthy of further research and discussion.

(1) At present, the stochastic model of various parameters and observations in multi-frequency and multi-GMSS PPP models is not sufficiently accurate, which can adversely affect the accuracy and reliability of parameter estimation. Therefore, it is necessary to further study stochastic models in multi-frequency and multi-GNSS PPP.

(2) Although the multi-frequency and multi-GNSS observations provide more abundant observational information, they present high dimensional ambiguity parameters and high correlation between the ambiguity parameters, which highlight challenges for the quality control of ambiguity resolution. High dimension and the high correlation between ambiguity parameters usually destroy the LAMBDA process, the efficiency becomes low, and the ratio test fails.(57) Establishing a reliable quality control system of ambiguity with high dimensionality and high correlation requires further discussion.

(3) Although the use of multi-frequency and multi-system GNSS observations considerably reduces the first initialization time of PPP AR and improves the positioning accuracy, the problem of instantaneous PPP AR has still not been solved at its origin, and the methods are still unable to provide global seamless instantaneous centimeter-level location services for single machine users. Other enhancement methods should be considered. Fortunately, the rapid development of low-orbit satellites in recent years provides the possibility of fundamentally solving this problem. Because of the high operational speeds of low-orbit satellites, the geometric structure among receiver satellites will change rapidly, which will significantly improve the convergence speed of position parameters. At present, some

relevant papers have used simulated low-orbit satellite observation data to enhance PPP and achieve instantaneous convergence of PPP.(58–59) Therefore, using real low-orbit satellite observational data to enhance PPP AR will be of intense research interest in the next few years.

(4) Even if PPP AR can achieve instantaneous convergence, it cannot provide continuous, reliable, and stable centimeter-level location services for global users in any environment, especially in underground, tunnel, urban dense building clusters, and other settings, a fact which is mainly determined by the vulnerability of the GNSS signal itself (such as the ease with which it can be interfered, spoofed, or otherwise compromised). These environments, however, are popular locations. Therefore, if professional PPP AR technology is applied to the popular location services, it must be combined with other sensors to overcome the defects of GNSS signals. In recent years, the development of emerging technologies such as visual positioning, 5G signals, low-cost LIDAR, and low-cost sensors has brought new opportunities for the ground enhancement of PPP AR. Based on existing PPP AR theory, error processing methods, and the parameter estimation method, the optimal integration of observational information provided by these new technologies to solve the continuity, reliability, stability, and other problems of location parameter estimation will be an important area of research going forward.

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