

# Optimization of Measurement Station Placement for Large-scale 3D Control Network Based on Collision Model and Simplex Algorithm

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Large-scale 3D control networks, which are essential for the precise measurement of large scientific facilities, have traditionally relied on empirical models. However, such approaches constrain the accuracy of measurement station placement and lack systematic optimization strategies. In this study, we systematically investigate the optimization of measurement station placement in complex environments and propose a method that integrates a collision model with the simplex algorithm. Simulation results indicate that, relative to conventional empirical model layouts, the proposed approach reduces station redundancy and deployment time, increases the number of effective measuring points by 8%, enhances single-point accuracy by 66%, and significantly improves the overall accuracy of the control network.

## 1. Introduction

Precise measurements of large scientific facilities, owing to their complex environments and high precision requirements, impose stringent demands on the control network during the measurement process.<sup>(1)</sup> The large 3D control network used in measurements typically relies on an empirical model of measurement station placement. However, owing to significant variations in different measurement scenarios, the empirical model is not universally applicable and provides only theoretical guidance. Therefore, it is essential to investigate effective 3D control network layout and optimization methods to enhance both the reliability and economic efficiency of the network.<sup>(2)</sup>

Network measurement station placement optimization involves a comprehensive planning of the number and location of measurement systems in the 3D control network, on the premise of ensuring the measurement accuracy, and taking into account many factors such as practicability, measurement cost, and efficiency.<sup>(3)</sup> Initially, the direct analysis method was used to analyze the

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specific placement and solve the extreme value points, but it was not widely used because of its excessive amount of computation and complex calculation process.<sup>(4)</sup> Scholars at home and abroad have conducted in-depth research on this. Takatsuji *et al.* used the matrix analysis method to analyze the measurement errors of different placements of the laser tracker and demonstrated the unreasonable system layout and accuracy evaluation indicators.<sup>(5)</sup> Ren *et al.* constructed the evaluation function of the measurement network placement of the laser tracking instrument and studied the network placement optimization method of combining global optimization with local search using a genetic algorithm.<sup>(6)</sup> Suthunyanakit *et al.* used the 3D model of the body interference method to determine whether a building blocks the measurement line.<sup>(7)</sup> Liu *et al.* judged the relationship between the measurement line and each surface of the building from the intersection of a line and plane, and detected the invisible measurement line.<sup>(8)</sup> Zhu *et al.* projected the cone and line of sight of the measurement point to the obstacle to the plane, and eliminated the nonvisible measurement line through the intersection of points and surfaces.<sup>(9)</sup> The existing network optimization methods have relatively simple measurement environments so the network optimization accuracy for complex environments is low because they lack systematic and visible network optimization models.<sup>(10)</sup>

Therefore, with the help of the digital model of the building environment, the simulation placement of the control network measurement points, and the results of simulated measurement, the network layout can be optimized to provide good guidance and reference for actual layouts.<sup>(11,12)</sup> We systematically studied the network optimization of a large 3D control network in large-scale scientific engineering measurement. Referring to the empirical model construction mode, we proposed a simplex network optimization algorithm based on collision detection and verified the accuracy of the simulation.

## 2. Linear Collision Model Based on Simplex Optimization Algorithm

The building environment structure in large-scale scientific engineering is special and complex, and there is a great overlap of the effective measurement areas of adjacent stations. The higher the number of measurement points in a single station, the more repeated observations in a single measurement point and the higher the network reliability. The area of each instrument to be measured is divided in accordance with the effective measurement range of the internal structure of the building and the instrument and the reference information given by the empirical model. Then, the placement of the station is optimized using the optimization algorithm of the station and the collision analysis model of the building and the line.

### 2.1 Determining vertex positions of simplex

Because of the effects of the location of the measurement stations and the number of measurements, there is a certain measurement error at each measurement point. When the measurement is carried out using an empirical model, the resulting measurement error follows a normal distribution. Therefore, the objective function  $f(p)$  is set to minimize the weighted sum of squares of measurement errors. An optimization algorithm is used to continuously adjust the

weights of the measurement points with the aim of obtaining a better measurement station placement for the measurement stations.

$$f(p) = V^T p V \quad (1)$$

In the formula,  $p$  represents the weight of each coordinate component of the measurement points in the matrix and  $V$  represents the measurement error of the point position.

If there are  $n$  measurement points within the measurement area, establish an optimization model with the objective of minimizing the weighted sum of squares of measurement errors.

$$\min = f(p) p \in \Omega \quad (2)$$

In the formula,  $\Omega = [0, 1]$  represents the range of values for the elements of the  $p$ -array.

To achieve this optimal solution, optimization algorithms are required. The simplex algorithm is a search algorithm that uses multidimensional space to find the optimal value of the objective function. It has good local optimization ability, simple calculation, and fast convergence.<sup>(13)</sup> If there are  $n$  measurement points in the measurement area, use these  $n$  measurement points as vertices to construct a simplex. That is, arrange the objective functions of each fixed point in the simplex in ascending order, obtain a series of vector points  $\{P_0, P_1, P_2, \dots, P_n\}$  and corresponding function values  $\{f(P_0), f(P_1), f(P_2), \dots, f(P_n)\}$ , and construct mapping points. The expression is

$$P_R = \bar{P} + \alpha(\bar{P} - P_n) \quad (3)$$

In the formula,  $\bar{P}$  is the average vector point and  $\alpha$  is the weight value used to calculate the mapping point  $P_R$ , usually set to 1. Determine the position of the vertices of the new simplex based on the relationship between the function values corresponding to each vector point and the function values corresponding to the mapping point  $P_R$ . The specific steps are shown in Table 1.

After the construction of the new simplex is completed, the objective function of each vertex of the simplex is calculated again, and a new round of optimization search begins. During the iteration, the vertices of the simplex continuously approach the minimum point of the objective function until the algorithm satisfies the convergence condition. At this point, obtain the optimal station placement.

Table 1  
Steps for determining vertex positions of simplex.

a) If $f(P_0) < f(P_R) < f(P_n)$ , move $P_n$ to $P_R$ .
b) If $f(P_R) < f(P_0)$ , structure expansion point $P_E = P_R + \beta(P_R - \bar{P})$ ; if $f(P_E) < f(P_R)$ , move $P_n$ to $P_E$ ; otherwise, move it to $P_R$ .
c) If $f(P_i) < f(P_R) < f(P_n)$ , move $P_n$ to $P_R$ ; otherwise, structure compression point $P_C = \bar{P} + \gamma(P_n - \bar{P})$ ; if $f(P_C) < f(P_n)$ , move $P_n$ to $P_C$ ; otherwise, all vectors are compressed along $P_0$ , resulting in $P_i = (1 - \rho)P_0 + \rho P_i$ ( $i = 0, 1, 2, \dots, n$ ).

### 2.2 Collision model of line visibility detection

For line visibility detection, the first step is to construct geometric topology information of the measurement site environment. According to the environmental digital model, extract the vertex coordinates of the internal environment and instrument equipment, and select any three vertex positions on the same object but not on the same plane to construct a rectangular prism. Extract the two longest diagonals of the rectangular prism and convert the collision detection of measuring lines into the problem of intersecting lines in computer graphics, as shown in Fig. 1.

Given that the origin of the measurement station is  $O = (X_i, Y_i, Z_i)$  and the measurement point is  $P = (X_j, Y_j, Z_j)$ , the vertices of the diagonal AB are  $A = (x_A, y_A, z_A)$  and  $B = (x_B, y_B, z_B)$ . The specific intersection can be determined by the following equations, taking the projection onto the  $Oxy$  plane as an example.

(1) The measurement line and the diagonal line intersect at a certain point; then,

$$\begin{cases} \left| \overline{PA} \times \overline{PO} \right| \cdot \left| \overline{PB} \times \overline{PO} \right| = \begin{vmatrix} x_A - X_j & y_A - Y_j \\ X_i - X_j & Y_i - Y_j \end{vmatrix} \cdot \begin{vmatrix} x_B - X_j & y_B - Y_j \\ X_i - X_j & Y_i - Y_j \end{vmatrix} \leq 0 \\ \left| \overline{AP} \times \overline{AB} \right| \cdot \left| \overline{AO} \times \overline{AB} \right| = \begin{vmatrix} X_j - x_A & Y_j - y_A \\ x_B - x_A & y_B - y_A \end{vmatrix} \cdot \begin{vmatrix} X_i - x_A & Y_i - y_A \\ x_B - x_A & y_B - y_A \end{vmatrix} \leq 0 \end{cases} \quad (4)$$

(2) The measurement line is collinear with a certain diagonal line (and has an overlapping part); then,

$$\begin{cases} \left| \overline{OB} \times \overline{BA} \right| = \begin{vmatrix} x_B - X_i & y_B - Y_i \\ x_A - x_B & y_A - y_B \end{vmatrix} = 0 \\ \max(X_i, X_j) > \min(x_A, x_B) \quad \max(Y_i, Y_j) > \min(y_A, y_B) \end{cases} \quad (5)$$

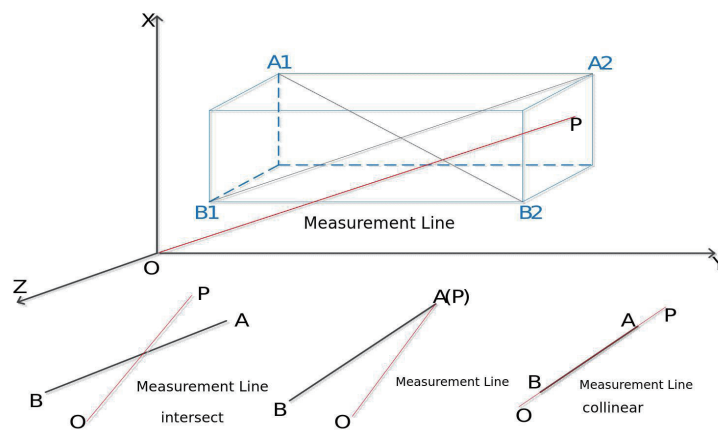


Fig. 1. (Color online) Line collision detection.

Similarly, when projected onto planes  $Oxz$  and  $Oyz$ , it can be proven that when the longest diagonal extracted from the constructed rectangular prism intersects or overlaps with the measurement line from the measurement station to the measurement point when projected onto a plane, the measurement line is obstructed and the instrument at the measurement station cannot measure the measurement point. Then, this measurement line should be removed.

The simplex algorithm and collision model successively calculate the best placement of each station and the most measurement times, and the systematic and visible network placement optimization is completed.

### 3. Test and Verification

#### 3.1 Simulation design

Mimic American NIF engineering built a digital model of a 3D simulation environment. The model consists of two laser halls of  $200 \times 33 \times 15 \text{ m}^3$  and a target range of  $35.5 \times 99 \times 15 \text{ m}^3$ . The specific model structure is shown in Fig. 2.

According to the empirical model, the measurement points are evenly laid out. The measurement points laid on the ground are about 10 m apart and the points on the shear wall are laid about 2 m from the ground. Because of the effective measurement range of the laser tracker and the shielding of the building environment, the instrument is set up about 1.5 m from the ground to make it at a similar height to the measurement point to observe as many measurement points as possible. To ensure that each measurement point can be measured by instruments at two or more measuring stations and that there are enough common points for coordinate conversion, at least two measuring stations are arranged within the inclusive range of each shear wall, as shown in Fig. 3.

Optimize and analyze the location of the measuring station, which is mainly affected by the position of the shear walls and measuring points. Extract the vertex coordinates of the instrument deployment area as the initial coordinates of the measurement station. Taking Leica AT960 as an example, its effective measurement distance is 80 m, and the ranging error increases with distance. To ensure accuracy, measurement points within 50 m of the station are selected. Optimize the selection of measurement stations using the simplex algorithm, and then conduct the visibility analysis of the measurement stations, removing measurement lines that pass through buildings, to obtain effective measurement data for each station, as shown in Fig. 4.

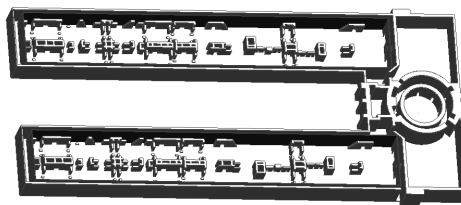


Fig. 2. Building body model.

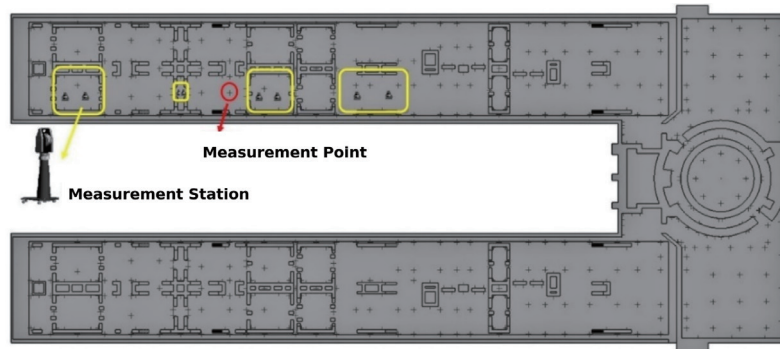


Fig. 3. (Color online) Placement diagram of measurement points.

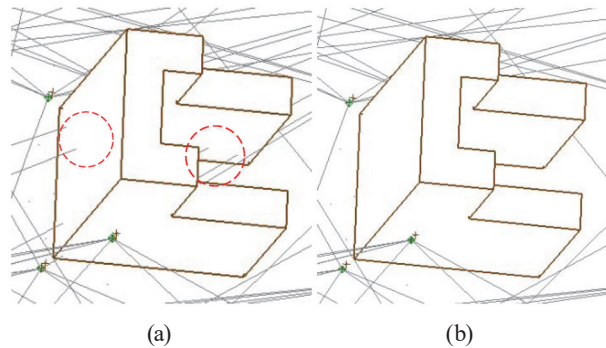


Fig. 4. (Color online) Line collision detection. (a) Collision line detection and (b) removal.

After the network optimization, the simulation measurement value is established in accordance with the theoretical measurement value, and the whole network solution calculation and accuracy evaluation are conducted.

### 3.2 Test results

The station placement was optimized by the network optimization method. The simplex convergence optimal value was obtained after 19 iterations, and the optimal station positions were obtained, as shown in Fig. 5. After optimization, the number of measurement stations was reduced from 116 to 113, the positions of measuring stations were changed, and the network layout time was reduced by 50% compared with the experienced network mode.

The numbers of stations observed at each measurement point before and after optimization are shown in Fig. 6. Of the 689 measurement points, 307 measurement points, or 45%, had more repeated measurements than before optimization. As seen from the intersection accuracy of the 3D coordinates of the laser tracker, the accuracy improvement is slow after the number of measurement points reaches six; thus, six measurement points can be taken as the critical number for evaluating the intersection accuracy. Similarly, when the measurement point is measured by six measurement stations, the measurement accuracy can reach a relatively stable

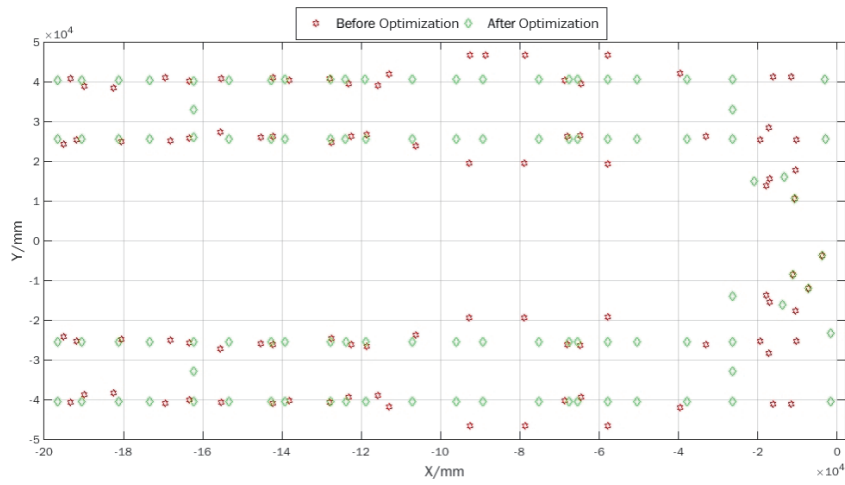
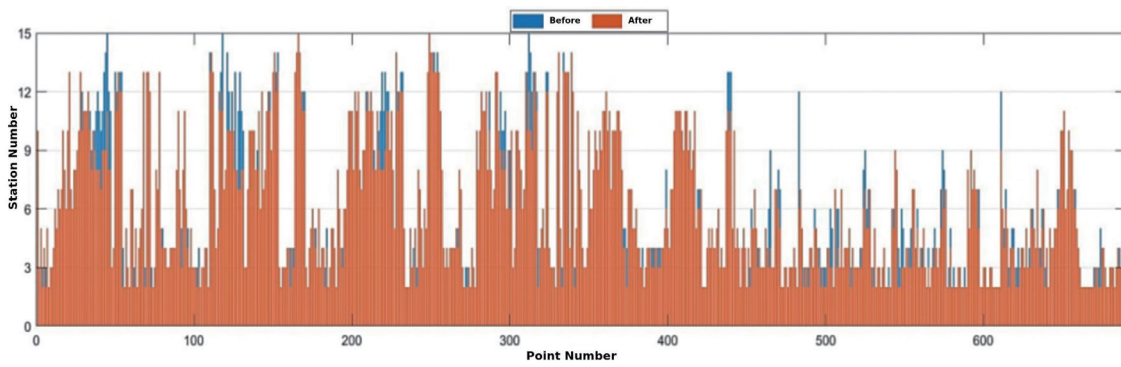
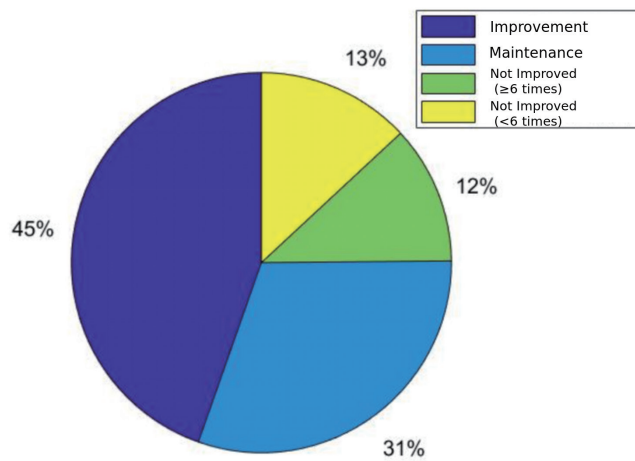


Fig. 5. (Color online) Placement of measurement stations.



(a)



(b)

Fig. 6. (Color online) Measurement optimization of measurement points. (a) Number of tests at a single point and (b) optimization of the number of times a single point is measured.

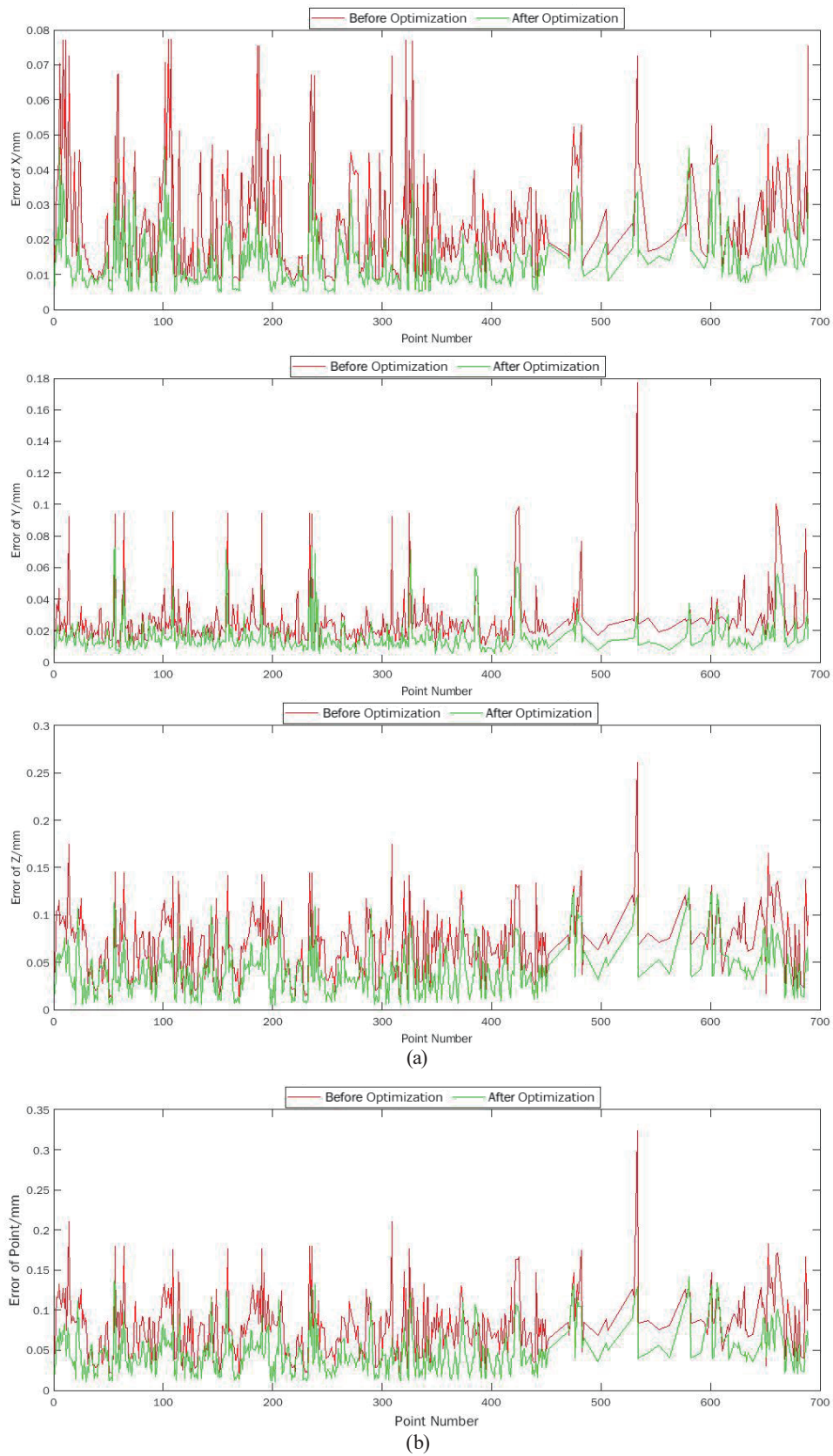


Fig. 7. (Color online) Analysis results of the solution accuracy of the 3D control network. (a) Coordinate uncertainty deviation and (b) point uncertainty.

value. The number of points that did not significantly improve the number of measurement points was 90, which accounts for 12% of the total.

After constructing simulation measurement values for the control network before and after optimization and performing overall network adjustment calculation, the coordinate covariance matrix  $u_{xyz}$  and point uncertainty  $u_{3d}$  were obtained, as shown in Fig. 7.

Before the optimization, the empirical model had 546 points satisfying the desired accuracy, and the point coverage rate was 79%. After the optimization, 594 points met the accuracy requirements after settlement, and the point coverage rate was 87%. The direction deviation of the point optimization data in the three directions and the point error of XYZ are much larger than those after optimization, and the uncertainty of the optimized point is less than 0.15 mm. The optimized solution accuracy of the 3D control network is improved from 0.13 to 0.10 mm and the accuracy of 66% single point uncertainty is an improvement. Therefore, the 3D control network layout scheme with optimized measurement station placement basically meets the accuracy requirements of the 3D control network layout of general large-scale scientific engineering.

#### 4. Conclusions

In large-scale scientific engineering surveys, the layout of the large-scale 3D control network plays a fundamental and key role in the precise implementation of the project. Traditional deployment methods mainly rely on empirical models, which have problems such as the difficulty of improving the accuracy of network deployment and the lack of fine system deployment methods, resulting in redundant observation stations, long network deployment time, few effective measurement points, and the insufficient accuracy of single-point and whole network solutions. In view of the above problems, we carried out a systematic study and proposed a large-scale 3D control network station optimization method based on a collision model and simplex algorithm. This method shows significant advantages in simulation experiments. Compared with the traditional empirical model deployment method, it effectively reduces the redundancy of observation stations and network deployment time, successfully increases the number of effective measurement points by 8%, improves the accuracy of single point measurement to 66%, and achieves a qualitative leap in the accuracy of the whole network solution. This research result provides new theoretical and technical support for the layout of a large-scale 3D control network in a complex environment, and is expected to promote the development of large-scale scientific engineering survey in a more efficient and accurate direction and provide a useful reference for subsequent related research and application, which can further expand and improve the applicability and optimization effect in different complex environments, so as to meet the growing demand for high-precision engineering survey.

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