

## Analysis of Thin Rod Acoustic Wave Gravimetric Sensors

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An analysis based on a perturbation theory for evaluating the mass sensitivity,  $S_m$ , for acoustic wave probes of gravimetric sensors made of thin rods is presented. The lowest-order flexural  $F_{11}$ , torsional  $T_{00}$ , and longitudinal  $L_{01}$  acoustic modes are of interest.  $S_m^V = \lim_{\Delta m_s \rightarrow 0} (1/V_0)(\Delta V/\Delta m_s)$  is found to be  $-1/2\rho_s a$ ,  $-2/\rho_s a$  and  $-1/\rho_s a$  for  $F_{11}$ ,  $T_{00}$  and  $L_{01}$  modes, respectively, where  $\Delta m_s$  is the uniformly distributed mass per unit area added to the surface of the rod;  $\Delta V = (V - V_0)$ , where  $V_0$  and  $V$  are the unloaded and loaded phase velocities, respectively;  $\rho_s$  is the density and  $a$  is the radius of the rod. We show that  $S_m^V = S_m^f V/V_g$ , where  $V_g$  is the group velocity, and  $S_m^f = \lim_{\Delta m_s \rightarrow 0} (1/f_0)(\Delta f/\Delta m_s)$ , where  $\Delta f = f - f_0$ , of which  $f_0$  and  $f$  are the resonant frequencies for unperturbed and mass loaded cases, respectively. This analysis can also be applied to bulk, Rayleigh surface, lowest flexural  $A_0$  and shear horizontal  $SH_0$  plate mode probes of gravimetric sensors. For flexural wave devices immersed in liquids we show that the mass sensitivity of the  $F_{11}$  mode is reduced but the reduction is less than that of the  $A_0$  mode.

### 1. Introduction

Recently, research and development in the area of integrated acoustic gravimetric sensors based on bulk (BAW),<sup>(1)</sup> surface (SAW),<sup>(2)</sup> plate<sup>(3,4)</sup> and thin